Rule formats for bounded nondeterminism in Nominal SOS

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Motivation

SOS and rule formats for bounded nondeterminism

- Process algebrae and SOS for providing semantics of concurrent programming:
  A transition system specification (TSS) consists of inference rules that induce a labelled transition system (LTS).

$$p \xrightarrow{l} p'$$

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- Finiteness of the number of transitions from a given process.
  Finite branching: an LTS is finite branching iff for every p, the set $\{(l, p') \mid p \xrightarrow{l} p'\}$ is finite.

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- Rule format: easy-to-cehck conditions on a TSS that guarantee a property of the associated LTS.
- Nominal SOS: nominal techniques to deal with binders and scopes in a nice way.
  Require arbitrary terms as labels!

Rule format for finite branching [Fokkink and Vu, 2003]

Theorem (Finite branching)

Let R be a TSS. The LTS associated to R is finite branching if the following holds:

(i) *All variables in R are source-dependent (**bounded nondeterminism format**).*

(ii) *R has no unguarded recursion (**strict stratification**).*

(iii) *R is bounded (**uniformity** and **finitely inhabited** η**-types**).*

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Definition (η-type)

Let η be a map from terms to finite sets of terms. We say $\eta(t)$ is the support for t.

A rule $H/t \xrightarrow{l} t'$ has η-type $\langle t, \psi \rangle$ iff

$$\psi(u) = \{m \mid u \in \eta(t) \wedge \exists u'.\ u \xrightarrow{m} u' \in H\} \text{ is a finite set of labels.}$$

Example (Rules for choice in BPA)

$$\frac{x_0 \xrightarrow{c} x_0'}{x_0 + x_1 \xrightarrow{c} x_0' + x_1} \qquad \frac{x_1 \xrightarrow{c} x_1'}{x_0 + x_1 \xrightarrow{c} x_0 + x_1'}$$

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Bounded nondeterminism format:

$$\frac{\left\{ u_k \xrightarrow{m_k} u_k' \right\}}{t \xrightarrow{l} t'}$$

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Strict stratification:

$$\begin{aligned} S(c) &= 0 \\ S(p_0 + p_1) &= 1 + S(p_0) + S(p_1) \\ S(p_0 \cdot p_1) &= 1 + S(p_0) + S(p_1) \end{aligned}$$

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Uniformity and finitely inhabited η-types:

$$\eta(x_0 + x_1) = \{x_0, x_1\}$$

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The problem

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- With the existing rule format junk rules may produce false negatives:

$$\frac{}{g(l_1) \xrightarrow{l_1} g(l_1)} \qquad \frac{g^i(x) \xrightarrow{l_i} y}{f(x) \xrightarrow{l_i} y}\ , \quad i \in \mathbb{N}$$

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- Exsiting solution only for ground actions. Nominal SOS requires terms as labels.
  Early π-calculus [Cimini, Mousavi, Reniers and Gabbay, 2012] (excerpt):

$$\frac{x \xrightarrow{y \overset{\tau}{\mapsto} z} x' \qquad a\#z \qquad a\#y}{[a]x \xrightarrow{y \overset{\tau}{\mapsto} x} [a]x'} \qquad \frac{x \xrightarrow{b \overset{\tau}{\mapsto} c} y}{in(a,[b]x) \xrightarrow{in(a,c)} y}$$

$$\frac{x_1 \xrightarrow{out(a,b)} y_1 \qquad x_2 \xrightarrow{in(a,b)} y_2}{x_1||x_2 \xrightarrow{\tau} y_1||y_2}$$

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ψ-calculus, CHOCS, ...

Filtering junk rules

Detection of junk rules

$$\frac{}{g(l_1) \xrightarrow{l_1} g(l_1)} \qquad \frac{g^i(x) \xrightarrow{l_i} y}{f(x) \xrightarrow{l_i} y}, \quad i \in \mathbb{N}$$

Detection of junk rules

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$$\langle g(x), \emptyset\rangle \qquad \begin{array}{ll} \langle f(x), \{g(x) \mapsto \{l_1\}\}\rangle & (i = 1) \\ \langle f(x), \{g(x) \mapsto \emptyset\}\rangle & (i \neq 1) \end{array}$$

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The TSS is not in the format!

But for $i \neq 1$, $\{g^i(x)\} \not\subseteq \eta(f(x)) = \{g(x)\}$.

Observation 1

A rule $H/t \xrightarrow{l} t'$ should not have valid η-type if $H \not\subseteq \eta(t)$.

Detection of junk rules

$$\frac{}{g(l_1) \xrightarrow{l_1} g(l_1)} \qquad \frac{g(x) \xrightarrow{l_1} l_i}{f(x) \xrightarrow{l_1} l_i}, \quad i \in \mathbb{N}$$

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The TSS is not in the format!

But each instantiation of the rule template has a different l_i as target.

Observation 2

The map ψ of an η-type should also keep track of the targets of premisses.

Arbitrary terms as labels

Dyadic transformations

Reminiscent of *commitments* in [Milner, 1993] and *residual datatypes* in [Bengtson, 2010].

$$t \xrightarrow{l} t'$$

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$$\mathcal{D}(t \xrightarrow{l} t') \quad = \quad t \longrightarrow (l, t')$$

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Reminiscent of *commitments* in [Milner, 1993] and *residual datatypes* in [Bengtson, 2010].

$$\begin{aligned} \mathcal{D}(t \xrightarrow{l} t') &= t \longrightarrow (l, t') \\ &= (t, l) \longrightarrow t' \end{aligned}$$

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$$
\begin{aligned}
\mathcal{D}(t \xrightarrow{l} t') &= t \longrightarrow (l, t') \\
&= (t, l) \longrightarrow t' \\
&\quad \ldots
\end{aligned}
$$

Dyadic transformations

Definition (Dyadic transformations)

The $\mathcal{D}_k$-dyadic transformations ($1 \leq k \leq 6$) are given by:

- $\mathcal{D}_1(t \xrightarrow{l} t') = t \longrightarrow (l, t')$.
- $\mathcal{D}_2(t \xrightarrow{l} t') = t' \longleftarrow (l, t)$.
- $\mathcal{D}_3(t \xrightarrow{l} t') = l \uparrow (t, t')$.
- $\mathcal{D}_4(t \xrightarrow{l} t') = (t, l) \longrightarrow t'$.
- $\mathcal{D}_5(t \xrightarrow{l} t') = (t', l) \longleftarrow t$.
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The $\mathcal{D}_k^{prj}$-dyadic projections ($1 \leq k \leq 3$ and $prj \in \{\pi_1, \pi_2\}$) are given by:

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- $\mathcal{D}_2^{\pi_1}(t \xrightarrow{l} t') = t' \longleftarrow l$.
- $\mathcal{D}_3^{\pi_1}(t \xrightarrow{l} t') = l \uparrow t$.
- $\mathcal{D}_3^{\pi_2}(t \xrightarrow{l} t') = l \uparrow t'$.
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We write $\mathcal{D}_k^{prj}$ (where $prj \in \{id, \pi_1, \pi_2\}$) for any of the above.

For a triadic TSS R, we say R is $\mathcal{D}_k^{prj}$-finite iff $R' = \mathcal{D}_k^{prj}(R)$ and R' is finite branching.

Bounded nondeterminism properties

Definition

(i) $\mathcal{D}_1^{id}$-finite: $\forall p.\ \{(l, p') \mid p \xrightarrow{l} p'\}$ is finite.

(ii) $\mathcal{D}_2^{id}$-finite: $\forall p'.\ \{(p, l) \mid p \xrightarrow{l} p'\}$ is finite.

(iii) $\mathcal{D}_3^{id}$-finite: $\forall l.\ \{(p, p') \mid p \xrightarrow{l} p'\}$ is finite.

(iv) $\mathcal{D}_4^{id}$-finite: $\forall p.\forall l.\ \{p' \mid p \xrightarrow{l} p'\}$ is finite.

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(vii) $\mathcal{D}_1^{\pi_1}$-finite: $\forall p.\ \{l \mid \exists p'.p \xrightarrow{l} p'\}$ is finite.

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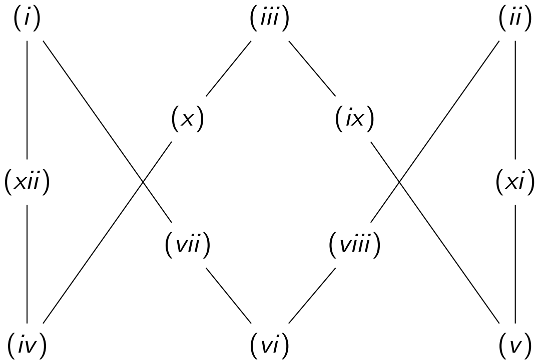
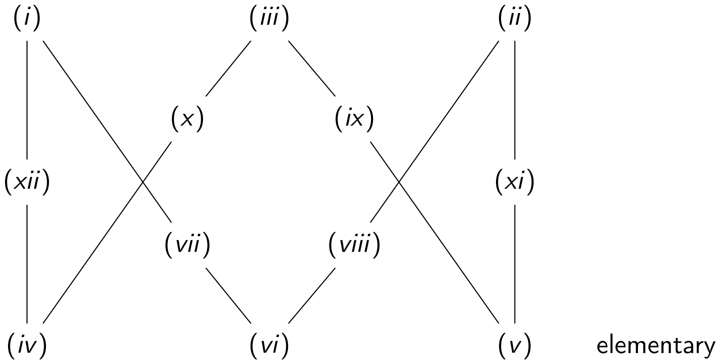
Bounded nondeterminism properties



Bounded nondeterminism properties



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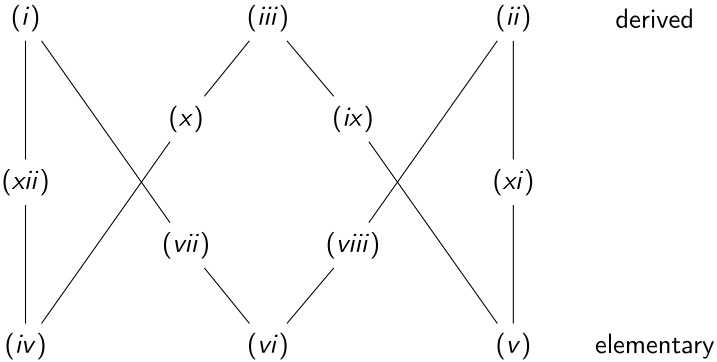

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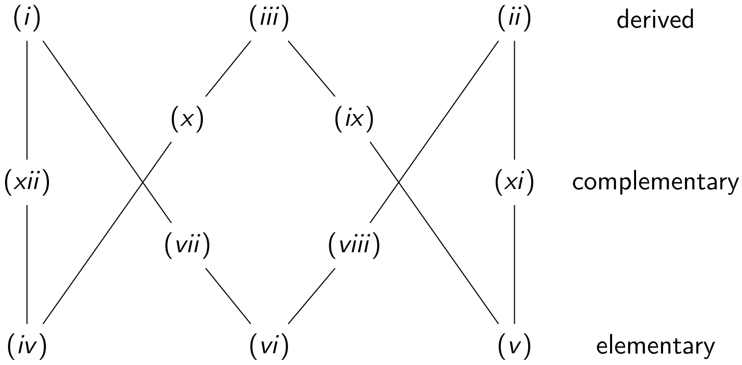

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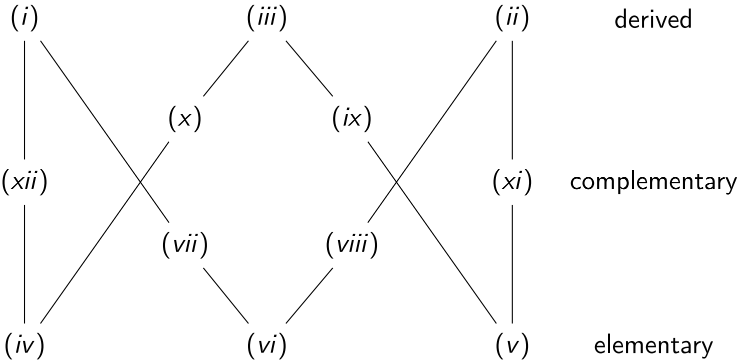
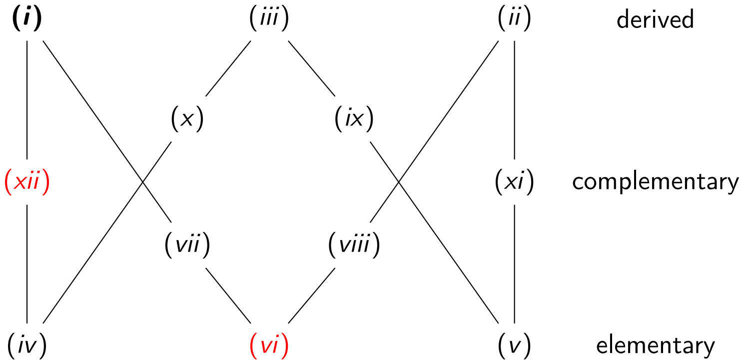


$$d \iff e \wedge c$$

Bounded nondeterminism properties



$$(i) \iff (xii) \wedge (vi)$$

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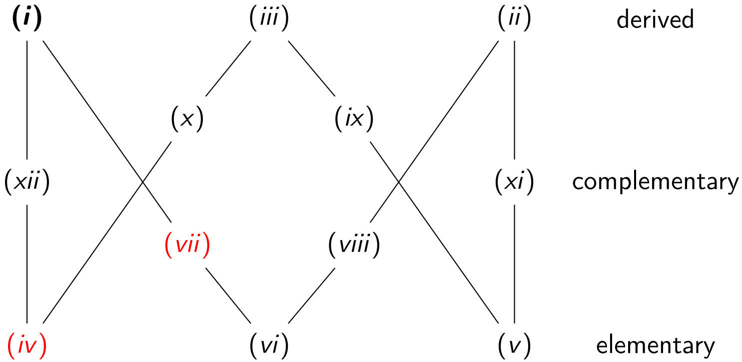

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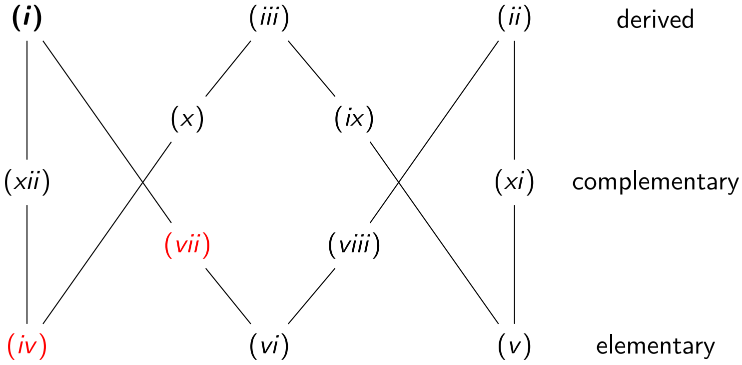

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(i.e., finite branching $\iff$ image finiteness $\wedge$ initials finiteness)

Our rule format

S-restricted support

Remember $\mathcal{D}_k^{prj}(t \xrightarrow{l} t') = s \longrightarrow r$

Definition (Partial strict stratification)

Let R be a dyadic TSS. S is a partial strict stratification of R iff the following conditions hold:

(i) $S(\sigma(s)) \neq \bot$, for every rule in R with source s and for every substitution σ that closes s,

(ii) For every rule in R with source s and set of premisses H, and for every $v \longrightarrow w \in H$, for each substitution σ that closes s and v such that $S(\sigma(v)) \neq \bot$, then $S(\sigma(v)) < S(\sigma(s))$.

S-restricted support

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Definition (Partial strict stratification)

Let R be a dyadic TSS. S is a partial strict stratification of R iff the following conditions hold:

(i) $S(\sigma(s)) \neq \perp$, for every rule in R with source s and for every substitution σ that closes s,

(ii) For every rule in R with source s and set of premisses H, and for every $v \longrightarrow w \in H$, for each substitution σ that closes s and v such that $S(\sigma(v)) \neq \perp$, then $S(\sigma(v)) < S(\sigma(s))$.

Definition (S-restricted support)

Let R be a dyadic TSS and S be a partial strict stratification of R. Let η be the map

$$\eta(s) = \{v \mid \exists\sigma.\, S(\sigma(v)) \neq \perp \wedge v \longrightarrow w \text{ is a premiss in a rule with source } s\}.$$

We say η is the S-restricted support map iff $\eta(s)$ is a finite set for each source s.

S-types and main theorem

Definition (S-types)

Let R be a dyadic TSS, S be a partial strict stratification of R, η be the associated S-restricted support map.
A rule $\rho = H/s \longrightarrow r$ has S-type $\langle s, \psi \rangle$ iff $\{v_i \mid i \in I\} \subseteq \eta(s)$ and

$$\psi(v) = \{w \mid v \in \eta(s) \wedge \exists w.\ v \longrightarrow w \in H\} \text{ is a finite set.}$$

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Theorem ($\mathcal{D}_k^{prj}$-finiteness)

Let R be a TSS with terms as labels and $R' = \mathcal{D}_k^{prj}(R)$. The LTS associated to R is $\mathcal{D}_k^{prj}$-finite if the following holds:

(i) *R' is in bounded nondeterminism format.*

(ii) *R' has a partial strict stratification S.*

(iii) *R' is uniform and has finitely inhabitted S-types.*

Applicability (CHOCS [Mousavi, Gabbay and Reniers, 2005])

$$p, q \quad ::= \quad 0 \mid a \mid c!x.p \mid c?a.p \mid (p \mid q) \mid \tau.p$$

$$\boxed{\xrightarrow{p}_{/a}} \qquad \frac{}{a \xrightarrow{z}_{/a} z} \qquad \frac{a \neq b}{b \xrightarrow{z}_{/a} b} \qquad \frac{x_0 \xrightarrow{z}_{/a} y_0 \qquad x_1 \xrightarrow{z}_{/a} y_1}{c!x_0.x_1 \xrightarrow{z}_{/a} c!y_0.y_1}$$

$$\frac{x \xrightarrow{z}_{/a} y \qquad a \neq b}{c?b.x \xrightarrow{z}_{/a} c?b.y} \qquad \frac{x_0 \xrightarrow{z}_{/a} y_0 \qquad x_1 \xrightarrow{z}_{/a} y_1}{(x_0 \mid x_1) \xrightarrow{z}_{/a} (y_0 \mid y_1)}$$

$$\boxed{\xrightarrow{p}_{c!}} \qquad \frac{}{c!x_0.x_1 \xrightarrow{x_0}_{c!} x_1} \qquad \frac{x_0 \xrightarrow{z}_{c!} y_0}{(x_0 \mid x_1) \xrightarrow{z}_{c!} (y_0 \mid x_1)}$$

$$\boxed{\xrightarrow{a}_{c?}} \qquad \frac{x_1 \xrightarrow{z}_{/a} y_1}{c?a.x_1 \xrightarrow{a}_{c?} y_1} \qquad \frac{x_0 \xrightarrow{z}_{c?} y_0}{(x_0 \mid x_1) \xrightarrow{z}_{c?} (y_0 \mid x_1)}$$

$$\boxed{\longrightarrow_{\tau}} \qquad \frac{x_0 \xrightarrow{z}_{c!} y_0 \qquad x_1 \xrightarrow{z}_{c?} y_1}{(x_0 \mid x_1) \longrightarrow_{\tau} (y_0 \mid y_1)} \qquad \frac{x_0 \longrightarrow_{\tau} y_0}{(x_0 \mid x_1) \longrightarrow_{\tau} (y_0 \mid x_1)} \qquad \frac{}{\tau.x \longrightarrow_{\tau} x}$$

Future work

- Refine the partial strict stratification by propagating the $\perp$ to the conclusions of the rules.
- Refine the bounded nondeterminism format to cover cases in which variables are discarded at some point.
- Extend the rule format to many-sorted signatures.

Summary

- Partial strict stratification and restricted support map for detecting more junk rules.
- Targets are important too.
- Dyadic transformations turn labels into a part of sources or targets.
- Space of bounded nondeterminism properties with rich structure.

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Thanks!

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