Rule formats for bounded nondeterminism in structural operational semantics

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Motivation

Structural operational semantics and bounded nondeterminism

A *transition system specification* (TSS) consists of inference rules that induce a *labelled transition system* (LTS) $\{p \xrightarrow{a} p'\}$

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Exercises 3.3 and 3.4 in *Semantics with Applications: An Appetizer* [Nielson and Nielson, 2007]

While language with nondeterminisitc choice and statement `random(x)`.

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x:=-1; while x<=0 do (x:=x-1 or x:=(-1)*x)
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Rule formats for finite branching: statically checkable (ideally) conditions on TSSs that guarantee continuous Scott-Strachey semantics ([Apt and Plotkin, 1986]).

Existing rule format for finite branching [Fokkink and Vu, 2003]

Theorem (Correctness of rule format)

Let R be a TSS. The LTS associated to R is finite branching if the following conditions hold:

(i) *R has no unguarded recursion (**strict stratification**).*

(ii) *Each rule in R gives rise to finitely many transitions from each process (**bounded nondeterminism format**).*

(iii) *Only finitely many rules in R can give rise to transitions from each process (**uniformity** and **finitely inhabited η-types**).*

Example (Rules for merge in BPA)

$$\cdots \qquad \frac{x_0 \xrightarrow{c} x_0'}{x_0 \| x_1 \xrightarrow{c} x_0' \| x_1} \qquad \frac{x_1 \xrightarrow{c} x_1'}{x_0 \| x_1 \xrightarrow{c} x_0 \| x_1'} \qquad \cdots$$

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Strict stratification:

$$\begin{array}{rcl} S(c) & = & 0 \\ S(p_0 \| p_1) & = & 1 + S(p_0) + S(p_1) \\ & & \cdots \end{array}$$

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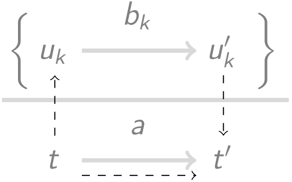
Bounded nondeterminism format:



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Uniformity and finitely inhabited η-types:

$$\eta(x_0 \| x_1) = \{x_0, x_1\}$$

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Uniformity and finitely inhabited η-types:

$$\langle x_0 \| x_1, \{x_0 \mapsto \{c\}, x_1 \mapsto \emptyset\}\rangle \qquad \langle x_0 \| x_1, \{x_0 \mapsto \emptyset, x_1 \mapsto \{c\}\}\rangle$$

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The problem

- Mechanising the proof of correctness of the rule format?

  ### Claim [Fokkink and Vu, 2003]

  For every term t there are finitely many maps ψ such that there exists a rule r of η-type $\langle t, \psi \rangle$ which gives rise to transitions.

  Proof: by assuming that the set of different maps ψ is infinite and deriving a contradiction. □

  Reasoning by contradiction here is not constructive!

- Bounded-nondeterminism properties other than finite branching?

  An LTS is *image finite* iff for every p and a the set $\{p' \mid p \xrightarrow{a} p'\}$ is finite.
  An LTS is *initials finite* iff for every p the set $\{a \mid \exists p'.p \xrightarrow{a} p'\}$ is finite.

  Rule formats for initials finiteness and for finite branching?

Our contribution

Constructive proof of correcteness of the rule format

For each process $p = \sigma(t)$, the ψ maps such that there exists a rule r of η-type $\langle t, \psi \rangle$ which gives rise to transitions are dependent functions of type $\psi : \Pi_{v \in \eta(t)} \{a \mid \sigma(v) \xrightarrow{a} q\}$.

Constructivity enables the mechanisation of the proof with a state-of-the-art proof assistant (work in progress).

Rule formats for image finiteness and initials finiteness

Definition (Image finiteness and initials finiteness)

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An LTS is *initials finite* iff for every p the set $\{a \mid \exists p'.p \xrightarrow{a} p'\}$ is finite.

The properties require modified η-types that either *ignore the targets* or *keep track of both actions and targets* in transitions.

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Example (Statement `random(x)`)

$$\frac{}{\langle \texttt{random}(x); S, s\rangle \xrightarrow{n} \langle S, s[x \mapsto n]\rangle}\ , \qquad n \in \mathbb{N}.$$

Related and Future work

- Generalise the rule formats to other bounded-nondeterminism properties [Aceto et al., 2016].
- Extend the rule formats to SOS with terms as labels [Aceto et al., 2016].
- Modify the rule formats to cover cases that we are aware are not covered yet.
- Extend the rule formats to many sorted signatures and Nominal SOS.

Summary

- Rule formats for bounded nondeterminism are useful to check whether a language admits a standard continuous semantics a la Scott-Strachey.
- We provide a constructive proof of correctness of the rule format for finite branching in [Fokkink and Vu, 2003].
- We provide rule formats for initials finiteness and image finiteness.

Summary

- Rule formats for bounded nondeterminism are useful to check whether a language admits a standard continuous semantics a la Scott-Strachey.
- We provide a constructive proof of correctness of the rule format for finite branching in [Fokkink and Vu, 2003].
- We provide rule formats for initials finiteness and image finiteness.

Happy Birthday to Hanne and Flemming!

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