All Rewrite Rules

From Event-B

CAUTION! Any modification to this page shall be announced on the User mailing list!

This page groups together all the rewrite rules implemented (or planned for implementation) in the Rodin prover. The rules themselves can be found in their respective location (for editing purposes):

Conventions used in these tables are described in The_Proving_Perspective_(Rodin_User_Manual)#Rewrite_rules

Contents

Set Rewrite Rules

Rules that are marked with a * in the first column are implemented in the latest version of Rodin. Rules without a * are planned to be implemented in future versions. Other conventions used in these tables are described in The_Proving_Perspective_(Rodin_User_Manual)#Rewrite_Rules.

	Name	Rule	Side Co
*	SIMP_SPECIAL_AND_BTRUE	$P \land \ldots \land \top \land \ldots \land Q \ \ \widehat{=}\ \ P \land \ldots \land Q$	
*	SIMP_SPECIAL_AND_BFALSE	$P \land \ldots \land \bot \land \ldots \land Q \ \ \widehat{=}\ \ \bot$	
*	SIMP_MULTI_AND	$P \land \ldots \land Q \land \ldots \land Q \land \ldots \land R \ \ \widehat{=}\ \ P \land \ldots \land Q \land \ldots \land R$	
*	SIMP_MULTI_AND_NOT	$P \land \ldots \land Q \land \ldots \land \neg Q \land \ldots \land R \ \ \widehat{=}\ \ \bot$	
*	SIMP_SPECIAL_OR_BTRUE	$P \lor \ldots \lor \top \lor \ldots \lor Q \ \ \widehat{=}\ \ \top$	
*	SIMP_SPECIAL_OR_BFALSE	$P \lor \ldots \lor \bot \lor \ldots \lor Q \ \ \widehat{=}\ \ P \lor \ldots \lor Q$	
*	SIMP_MULTI_OR	$P \lor \ldots \lor Q \lor \ldots \lor Q \lor \ldots \lor R \ \ \widehat{=}\ \ P \lor \ldots \lor Q \lor \ldots \lor R$	
*	SIMP_MULTI_OR_NOT	$P \lor \ldots \lor Q \lor \ldots \lor \neg Q \land \ldots \land R \ \ \widehat{=}\ \ \top$	
*	SIMP_SPECIAL_IMP_BTRUE_R	$P \Rightarrow \top \ \ \widehat{=}\ \ \top$	
*	SIMP_SPECIAL_IMP_BTRUE_L	$\top \Rightarrow P \ \ \widehat{=}\ \ P$	
*	SIMP_SPECIAL_IMP_BFALSE_R	$P \Rightarrow \bot \ \ \widehat{=}\ \ \neg P$	
*	SIMP_SPECIAL_IMP_BFALSE_L	$\bot \Rightarrow P \ \ \widehat{=}\ \ \top$	
*	SIMP_MULTI_IMP	$P \Rightarrow P \ \ \widehat{=}\ \ \top$	
*	SIMP_MULTI_IMP_NOT_L	$\neg P \Rightarrow P \ \ \widehat{=}\ \ P$	
*	SIMP_MULTI_IMP_NOT_R	$P \Rightarrow \neg P \ \ \widehat{=}\ \ \neg P$	
*	SIMP_MULTI_IMP_AND	$P \land \ldots \land Q \land \ldots \land R \Rightarrow Q \ \ \widehat{=}\ \ \top$	
*	SIMP_MULTI_IMP_AND_NOT_R	$P \land \ldots \land Q \land \ldots \land R \Rightarrow \neg Q \ \ \widehat{=}\ \ \neg (P \land \ldots \land Q \land \ldots \land R)$	
*	SIMP_MULTI_IMP_AND_NOT_L	$P \land \ldots \land \neg Q \land \ldots \land R \Rightarrow Q \ \ \widehat{=}\ \ \neg (P \land \ldots \land \neg Q \land \ldots \land R)$	
*	SIMP_MULTI_EQV	$P \Leftrightarrow P \ \ \widehat{=}\ \ \top$	
*	SIMP_MULTI_EQV_NOT	$P \Leftrightarrow \neg P \ \ \widehat{=}\ \ \bot$	
*	SIMP_SPECIAL_NOT_BTRUE	$\neg \top \ \ \widehat{=}\ \ \bot$	
*	SIMP_SPECIAL_NOT_BFALSE	$\neg \bot \ \ \widehat{=}\ \ \top$	
*	SIMP_NOT_NOT	$\neg \neg P \ \ \widehat{=}\ \ P$	
*	SIMP_NOTEQUAL	$E \neq F \ \ \widehat{=}\ \ \neg E = F$	
*	SIMP_NOTIN	$E \notin F \ \ \widehat{=}\ \ \neg E \in F$	

*	SIMP_NOTSUBSET	$E \not\subset F \;\;\widehat{=}\;\; \neg\, E \subset F$	
*	SIMP_NOTSUBSETEQ	$E \not\subseteq F \;\;\widehat{=}\;\; \neg\, E \subseteq F$	
*	SIMP_NOT_LE	$\neg\, a \leq b \;\;\widehat{=}\;\; a > b$	
*	SIMP_NOT_GE	$\neg\, a \geq b \;\;\widehat{=}\;\; a < b$	
*	SIMP_NOT_LT	$\neg\, a < b \;\;\widehat{=}\;\; a \geq b$	
*	SIMP_NOT_GT	$\neg\, a > b \;\;\widehat{=}\;\; a \leq b$	
*	SIMP_SPECIAL_NOT_EQUAL_FALSE_R	$\neg\,(E = \mathrm{FALSE}) \;\;\widehat{=}\;\; (E = \mathrm{TRUE})$	
*	SIMP_SPECIAL_NOT_EQUAL_FALSE_L	$\neg\,(\mathrm{FALSE} = E) \;\;\widehat{=}\;\; (\mathrm{TRUE} = E)$	
*	SIMP_SPECIAL_NOT_EQUAL_TRUE_R	$\neg\,(E = \mathrm{TRUE}) \;\;\widehat{=}\;\; (E = \mathrm{FALSE})$	
*	SIMP_SPECIAL_NOT_EQUAL_TRUE_L	$\neg\,(\mathrm{TRUE} = E) \;\;\widehat{=}\;\; (\mathrm{FALSE} = E)$	
*	SIMP_FORALL_AND	$\forall x \cdot P \wedge Q \;\;\widehat{=}\;\; (\forall x \cdot P) \wedge (\forall x \cdot Q)$	
*	SIMP_EXISTS_OR	$\exists x \cdot P \vee Q \;\;\widehat{=}\;\; (\exists x \cdot P) \vee (\exists x \cdot Q)$	
*	SIMP_EXISTS_IMP	$\exists x \cdot P \Rightarrow Q \;\;\widehat{=}\;\; (\forall x \cdot P) \Rightarrow (\exists x \cdot Q)$	
*	SIMP_FORALL	$\forall \ldots, z, \ldots \cdot P(z) \;\;\widehat{=}\;\; \forall z \cdot P(z)$	Quantified identifiers than z do r in P
*	SIMP_EXISTS	$\exists \ldots, z, \ldots \cdot P(z) \;\;\widehat{=}\;\; \exists z \cdot P(z)$	Quantified identifiers than z do r in P
*	SIMP_MULTI_EQUAL	$E = E \;\;\widehat{=}\;\; \top$	
*	SIMP_MULTI_NOTEQUAL	$E \neq E \;\;\widehat{=}\;\; \bot$	
*	SIMP_EQUAL_MAPSTO	$E \mapsto F = G \mapsto H \;\;\widehat{=}\;\; E = G \wedge F = H$	
*	SIMP_EQUAL_SING	$\{E\} = \{F\} \;\;\widehat{=}\;\; E = F$	
*	SIMP_SPECIAL_EQUAL_TRUE	$\mathrm{TRUE} = \mathrm{FALSE} \;\;\widehat{=}\;\; \bot$	
*	SIMP_TYPE_SUBSETEQ	$S \subseteq Ty \;\;\widehat{=}\;\; \top$	where Ty i expression
*	SIMP_SUBSETEQ_SING	$\{E\} \subseteq S \;\;\widehat{=}\;\; E \in S$	where E single expr
*	SIMP_SPECIAL_SUBSETEQ	$\varnothing \subseteq S \;\;\widehat{=}\;\; \top$	
*	SIMP_MULTI_SUBSETEQ	$S \subseteq S \;\;\widehat{=}\;\; \top$	
*	SIMP_SUBSETEQ_BUNION	$S \subseteq A \cup \ldots \cup S \cup \ldots \cup B \;\;\widehat{=}\;\; \top$	
*	SIMP_SUBSETEQ_BINTER	$A \cap \ldots \cap S \cap \ldots \cap B \subseteq S \;\;\widehat{=}\;\; \top$	
*	DERIV_SUBSETEQ_BUNION	$A \cup \ldots \cup B \subseteq S \;\;\widehat{=}\;\; A \subseteq S \wedge \ldots \wedge B \subseteq S$	
*	DERIV_SUBSETEQ_BINTER	$S \subseteq A \cap \ldots \cap B \;\;\widehat{=}\;\; S \subseteq A \wedge \ldots \wedge S \subseteq B$	
*	SIMP_SPECIAL_IN	$E \in \varnothing \;\;\widehat{=}\;\; \bot$	
*	SIMP_MULTI_IN	$B \in \{A, \ldots, B, \ldots, C\} \;\;\widehat{=}\;\; \top$	
*	SIMP_IN_SING	$E \in \{F\} \;\;\widehat{=}\;\; E = F$	
*	SIMP_MULTI_SETENUM	$\{A, \ldots, B, \ldots, B, \ldots, C\} \;\;\widehat{=}\;\; \{A, \ldots, B, \ldots, C\}$	
*	SIMP_SPECIAL_BINTER	$S \cap \ldots \cap \varnothing \cap \ldots \cap T \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_TYPE_BINTER	$S \cap \ldots \cap Ty \cap \ldots \cap T \;\;\widehat{=}\;\; S \cap \ldots \cap T$	where Ty i expression
*	SIMP_MULTI_BINTER	$S \cap \ldots \cap T \cap \ldots \cap T \cap \ldots \cap U \;\;\widehat{=}\;\; S \cap \ldots \cap T \cap \ldots \cap U$	
*	SIMP_MULTI_EQUAL_BINTER	$S \cap \ldots \cap T \cap \ldots \cap U = T \;\;\widehat{=}\;\; T \subseteq S \cap \ldots \cap U$	
*	SIMP_SPECIAL_BUNION	$S \cup \ldots \cup \varnothing \cup \ldots \cup T \;\;\widehat{=}\;\; S \cup \ldots \cup T$	
*	SIMP_TYPE_BUNION	$S \cup \ldots \cup Ty \cup \ldots \cup T \;\;\widehat{=}\;\; Ty$	where Ty i expression
*	SIMP_MULTI_BUNION	$S \cup \ldots \cup T \cup \ldots \cup T \cup \ldots \cup U \;\;\widehat{=}\;\; S \cup \ldots \cup T \cup \ldots \cup U$	

*	SIMP_MULTI_EQUAL_BUNION	$S \cup \ldots \cup T \cup \ldots \cup U = T \;\;\widehat{=}\;\; S \cup \ldots \cup U \subseteq T$	
*	SIMP_MULTI_SETMINUS	$S \setminus S \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_SPECIAL_SETMINUS_R	$S \setminus \varnothing \;\;\widehat{=}\;\; S$	
*	SIMP_SPECIAL_SETMINUS_L	$\varnothing \setminus S \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_TYPE_SETMINUS	$S \setminus Ty \;\;\widehat{=}\;\; \varnothing$	where Ty i expression
*	SIMP_TYPE_SETMINUS_SETMINUS	$Ty \setminus (Ty \setminus S) \;\;\widehat{=}\;\; S$	where Ty i expression
*	SIMP_KUNION_POW	$\mathrm{union}(\mathbb{P}(S)) \;\;\widehat{=}\;\; S$	
*	SIMP_KUNION_POW1	$\mathrm{union}(\mathbb{P}_1(S)) \;\;\widehat{=}\;\; S$	
*	SIMP_SPECIAL_KUNION	$\mathrm{union}(\{\varnothing\}) \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_SPECIAL_QUNION	$\bigcup x \cdot \bot \mid E \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_SPECIAL_KINTER	$\mathrm{inter}(\{\varnothing\}) \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_KINTER_POW	$\mathrm{inter}(\mathbb{P}(S)) \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_SPECIAL_POW	$\mathbb{P}(\varnothing) \;\;\widehat{=}\;\; \{\varnothing\}$	
*	SIMP_SPECIAL_POW1	$\mathbb{P}_1(\varnothing) \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_SPECIAL_CPROD_R	$S \times \varnothing \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_SPECIAL_CPROD_L	$\varnothing \times S \;\;\widehat{=}\;\; \varnothing$	
	SIMP_COMPSET_EQUAL	$\{x, y \cdot x = E(y) \wedge P(y) \mid F(x, y)\} \;\;\widehat{=}\;\; \{y \cdot P(y) \mid F(E(y), y)\}$	where x n in E and in P
*	SIMP_COMPSET_IN	$\{x \cdot x \in S \mid x\} \;\;\widehat{=}\;\; S$	where x n in S
*	SIMP_COMPSET_SUBSETEQ	$\{x \cdot x \subseteq S \mid x\} \;\;\widehat{=}\;\; \mathbb{P}(S)$	where x n in S
*	SIMP_SPECIAL_COMPSET_BFALSE	$\{x \cdot \bot \mid x\} \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_SPECIAL_COMPSET_BTRUE	$\{x \cdot \top \mid E\} \;\;\widehat{=}\;\; Ty$	where the t E is Ty a is a maplet combinatio locally-bou pairwise-d bound iden
*	SIMP_SUBSETEQ_COMPSET_L	$\{x \cdot P(x) \mid E(x)\} \subseteq S \;\;\widehat{=}\;\; \forall y \cdot P(y) \Rightarrow E(y) \in S$	where y i
*	SIMP_IN_COMPSET	$F \in \{x, y, \ldots \cdot P(x, y, \ldots) \mid E(x, y, \ldots)\} \;\;\widehat{=}\;\; \exists x, y, \ldots \cdot P(x, y, \ldots) \wedge E(x, y, \ldots) = F$	where x, are not fre
*	SIMP_IN_COMPSET_ONEPOINT	$E \in \{x \cdot P(x) \mid x\} \;\;\widehat{=}\;\; P(E)$	Equivalent general simplificat followed b Point Rule application the last co predicate
	SIMP_SUBSETEQ_COMPSET_R	$S \subseteq \{x \cdot P(x) \mid x\} \;\;\widehat{=}\;\; \forall y \cdot y \in S \Rightarrow P(y)$	where y r in $S, \{x \cdot P($
*	SIMP_SPECIAL_OVERL	$r \mathbin{<\!\!+} \ldots \mathbin{<\!\!+} \varnothing \mathbin{<\!\!+} \ldots \mathbin{<\!\!+} s \;\;\widehat{=}\;\; r \mathbin{<\!\!+} \ldots \mathbin{<\!\!+} s$	
*	SIMP_SPECIAL_KBOOL_BTRUE	$\mathrm{bool}(\top) \;\;\widehat{=}\;\; \mathrm{TRUE}$	
*	SIMP_SPECIAL_KBOOL_BFALSE	$\mathrm{bool}(\bot) \;\;\widehat{=}\;\; \mathrm{FALSE}$	
	DISTRI_SUBSETEQ_BUNION_SING	$S \cup \{F\} \subseteq T \;\;\widehat{=}\;\; S \subseteq T \wedge F \in T$	where F is expression
*	DEF_FINITE	$\mathrm{finite}(S) \;\;\widehat{=}\;\; \exists n, f \cdot f \in 1 \,..\, n \rightarrowtail\!\!\!\!\rightarrow S$	
*	SIMP_SPECIAL_FINITE	$\mathrm{finite}(\varnothing) \;\;\widehat{=}\;\; \top$	

*	SIMP_FINITE_SETENUM	$\mathrm{finite}(\{a, \ldots, b\}) \;\; \widehat{=} \;\; \top$	
*	SIMP_FINITE_BUNION	$\mathrm{finite}(S \cup T) \;\; \widehat{=} \;\; \mathrm{finite}(S) \land \mathrm{finite}(T)$	
	SIMP_FINITE_UNION	$\mathrm{finite}(\mathrm{union}(S)) \;\; \widehat{=} \;\; \mathrm{finite}(S) \;\land\; (\forall x \cdot x \in S \Rightarrow finite(x))$	
	SIMP_FINITE_QUNION	$\mathrm{finite}(\bigcup x \cdot P \mid E) \;\; \widehat{=} \;\; \mathrm{finite}(\{x \cdot P \mid E\}) \;\land\; (\forall x \cdot P \Rightarrow finite(E))$	
*	SIMP_FINITE_POW	$\mathrm{finite}(\mathbb{P}(S)) \;\; \widehat{=} \;\; \mathrm{finite}(S)$	
*	DERIV_FINITE_CPROD	$\mathrm{finite}(S \times T) \;\; \widehat{=} \;\; S = \varnothing \lor T = \varnothing \lor (\mathrm{finite}(S) \land \mathrm{finite}(T))$	
*	SIMP_FINITE_CONVERSE	$\mathrm{finite}(r^{-1}) \;\; \widehat{=} \;\; \mathrm{finite}(r)$	
*	SIMP_FINITE_UPTO	$\mathrm{finite}(a \mathbin{..} b) \;\; \widehat{=} \;\; \top$	
*	SIMP_FINITE_ID	$\mathrm{finite}(\mathrm{id}) \;\; \widehat{=} \;\; \mathrm{finite}(S)$	where id h $\mathbb{P}(S \times S$
*	SIMP_FINITE_ID_DOMRES	$\mathrm{finite}(E \lhd \mathrm{id}) \;\; \widehat{=} \;\; \mathrm{finite}(E)$	
*	SIMP_FINITE_PRJ1	$\mathrm{finite}(\mathrm{prj}_1) \;\; \widehat{=} \;\; \mathrm{finite}(S \times T)$	where prj type $\mathbb{P}(S \times T$
*	SIMP_FINITE_PRJ2	$\mathrm{finite}(\mathrm{prj}_2) \;\; \widehat{=} \;\; \mathrm{finite}(S \times T)$	where prj type $\mathbb{P}(S \times T$
*	SIMP_FINITE_PRJ1_DOMRES	$\mathrm{finite}(E \lhd \mathrm{prj}_1) \;\; \widehat{=} \;\; \mathrm{finite}(E)$	
*	SIMP_FINITE_PRJ2_DOMRES	$\mathrm{finite}(E \lhd \mathrm{prj}_2) \;\; \widehat{=} \;\; \mathrm{finite}(E)$	
*	SIMP_FINITE_NATURAL	$\mathrm{finite}(\mathbb{N}) \;\; \widehat{=} \;\; \bot$	
*	SIMP_FINITE_NATURAL1	$\mathrm{finite}(\mathbb{N}_1) \;\; \widehat{=} \;\; \bot$	
*	SIMP_FINITE_INTEGER	$\mathrm{finite}(\mathbb{Z}) \;\; \widehat{=} \;\; \bot$	
*	SIMP_FINITE_BOOL	$\mathrm{finite}(\mathrm{BOOL}) \;\; \widehat{=} \;\; \top$	
*	SIMP_FINITE_LAMBDA	$\mathrm{finite}(\{x \cdot P \mid E \mapsto F\}) \;\; \widehat{=} \;\; \mathrm{finite}(\{x \cdot P \mid E\})$	where E maplet combinatic bound iden and expres that are no by the comprehen (i.e., E is syntactical injective) a identifiers by the comprehen that occur also occur
*	SIMP_TYPE_IN	$t \in Ty \;\; \widehat{=} \;\; \top$	where Ty i expression
*	SIMP_SPECIAL_EQV_BTRUE	$P \Leftrightarrow \top \;\; \widehat{=} \;\; P$	
*	SIMP_SPECIAL_EQV_BFALSE	$P \Leftrightarrow \bot \;\; \widehat{=} \;\; \neg P$	
*	DEF_SUBSET	$A \subset B \;\; \widehat{=} \;\; A \subseteq B \land \neg A = B$	
*	SIMP_SPECIAL_SUBSET_R	$S \subset \varnothing \;\; \widehat{=} \;\; \bot$	
*	SIMP_SPECIAL_SUBSET_L	$\varnothing \subset S \;\; \widehat{=} \;\; \neg\, S = \varnothing$	
*	SIMP_TYPE_SUBSET_L	$S \subset Ty \;\; \widehat{=} \;\; S \neq Ty$	where Ty i expression
*	SIMP_MULTI_SUBSET	$S \subset S \;\; \widehat{=} \;\; \bot$	
*	SIMP_EQUAL_CONSTR	$\mathrm{constr}(a_1, \ldots, a_n) = \mathrm{constr}(b_1, \ldots, b_n) \;\; \widehat{=} \;\; a_1 = b_1 \land \ldots \land a_n = b_n$	where cor datatype constructo
*	SIMP_EQUAL_CONSTR_DIFF	$\mathrm{constr}_1(\ldots) = \mathrm{constr}_2(\ldots) \;\; \widehat{=} \;\; \bot$	where cor and const different d constructo

*	SIMP_DESTR_CONSTR	$\mathrm{destr}(\mathrm{constr}(a_1, \ldots, a_n)) \ \widehat{=}\ a_i$	where des datatype de for the i-th argument c datatype constructor constr
*	DISTRI_AND_OR	$P \land (Q \lor R) \ \widehat{=}\ (P \land Q) \lor (P \land R)$	
*	DISTRI_OR_AND	$P \lor (Q \land R) \ \widehat{=}\ (P \lor Q) \land (P \lor R)$	
*	DEF_OR	$P \lor Q \lor \ldots \lor R \ \widehat{=}\ \neg P \Rightarrow (Q \lor \ldots \lor R)$	
*	DERIV_IMP	$P \Rightarrow Q \ \widehat{=}\ \neg Q \Rightarrow \neg P$	
*	DERIV_IMP_IMP	$P \Rightarrow (Q \Rightarrow R) \ \widehat{=}\ P \land Q \Rightarrow R$	
*	DISTRI_IMP_AND	$P \Rightarrow (Q \land R) \ \widehat{=}\ (P \Rightarrow Q) \land (P \Rightarrow R)$	
*	DISTRI_IMP_OR	$(P \lor Q) \Rightarrow R \ \widehat{=}\ (P \Rightarrow R) \land (Q \Rightarrow R)$	
*	DEF_EQV	$P \Leftrightarrow Q \ \widehat{=}\ (P \Rightarrow Q) \land (Q \Rightarrow P)$	
*	DISTRI_NOT_AND	$\neg (P \land Q) \ \widehat{=}\ \neg P \lor \neg Q$	
*	DISTRI_NOT_OR	$\neg (P \lor Q) \ \widehat{=}\ \neg P \land \neg Q$	
*	DERIV_NOT_IMP	$\neg (P \Rightarrow Q) \ \widehat{=}\ P \land \neg Q$	
*	DERIV_NOT_FORALL	$\neg \forall x \cdot P \ \widehat{=}\ \exists x \cdot \neg P$	
*	DERIV_NOT_EXISTS	$\neg \exists x \cdot P \ \widehat{=}\ \forall x \cdot \neg P$	
*	DEF_IN_MAPSTO	$E \mapsto F \in S \times T \ \widehat{=}\ E \in S \land F \in T$	
*	DEF_IN_POW	$E \in \mathbb{P}(S) \ \widehat{=}\ E \subseteq S$	
*	DEF_IN_POW1	$E \in \mathbb{P}_1(S) \ \widehat{=}\ E \in \mathbb{P}(S) \land S \neq \varnothing$	
*	DEF_SUBSETEQ	$S \subseteq T \ \widehat{=}\ \forall x \cdot x \in S \Rightarrow x \in T$	where x is in S or T
*	DEF_IN_BUNION	$E \in S \cup T \ \widehat{=}\ E \in S \lor E \in T$	
*	DEF_IN_BINTER	$E \in S \cap T \ \widehat{=}\ E \in S \land E \in T$	
*	DEF_IN_SETMINUS	$E \in S \setminus T \ \widehat{=}\ E \in S \land \neg (E \in T)$	
*	DEF_IN_SETENUM	$E \in \{A, \ldots, B\} \ \widehat{=}\ E = A \lor \ldots \lor E = B$	
*	DEF_IN_KUNION	$E \in \mathrm{union}(S) \ \widehat{=}\ \exists s \cdot s \in S \land E \in s$	where s is
*	DEF_IN_QUNION	$E \in (\bigcup x \cdot P(x) \mid T(x)) \ \widehat{=}\ \exists s \cdot P(s) \land E \in T(s)$	where s is
*	DEF_IN_KINTER	$E \in \mathrm{inter}(S) \ \widehat{=}\ \forall s \cdot s \in S \Rightarrow E \in s$	where s is
*	DEF_IN_QINTER	$E \in (\bigcap x \cdot P(x) \mid T(x)) \ \widehat{=}\ \forall s \cdot P(s) \Rightarrow E \in T(s)$	where s is
*	DEF_IN_UPTO	$E \in a \,..\, b \ \widehat{=}\ a \leq E \land E \leq b$	
*	DISTRI_BUNION_BINTER	$S \cup (T \cap U) \ \widehat{=}\ (S \cup T) \cap (S \cup U)$	
*	DISTRI_BINTER_BUNION	$S \cap (T \cup U) \ \widehat{=}\ (S \cap T) \cup (S \cap U)$	
	DISTRI_BINTER_SETMINUS	$S \cap (T \setminus U) \ \widehat{=}\ (S \cap T) \setminus (S \cap U)$	
	DISTRI_SETMINUS_BUNION	$S \setminus (T \cup U) \ \widehat{=}\ S \setminus T \setminus U$	
*	DERIV_TYPE_SETMINUS_BINTER	$Ty \setminus (S \cap T) \ \widehat{=}\ (Ty \setminus S) \cup (Ty \setminus T)$	where Ty i expression
*	DERIV_TYPE_SETMINUS_BUNION	$Ty \setminus (S \cup T) \ \widehat{=}\ (Ty \setminus S) \cap (Ty \setminus T)$	where Ty i expression
*	DERIV_TYPE_SETMINUS_SETMINUS	$Ty \setminus (S \setminus T) \ \widehat{=}\ (Ty \setminus S) \cup T$	where Ty i expression
	DISTRI_CPROD_BINTER	$S \times (T \cap U) \ \widehat{=}\ (S \times T) \cap (S \times U)$	
	DISTRI_CPROD_BUNION	$S \times (T \cup U) \ \widehat{=}\ (S \times T) \cup (S \times U)$	
	DISTRI_CPROD_SETMINUS	$S \times (T \setminus U) \ \widehat{=}\ (S \times T) \setminus (S \times U)$	
			where $\mathbb{P}($

*	DERIV_SUBSETEQ	$S \subseteq T \;\;\widehat{=}\;\; (Ty \setminus T) \subseteq (Ty \setminus S)$	the type of T
*	DERIV_EQUAL	$S = T \;\;\widehat{=}\;\; S \subseteq T \land T \subseteq S$	where $\mathbb{P}($ the type of T
*	DERIV_SUBSETEQ_SETMINUS_L	$A \setminus B \subseteq S \;\;\widehat{=}\;\; A \subseteq B \cup S$	
*	DERIV_SUBSETEQ_SETMINUS_R	$S \subseteq A \setminus B \;\;\widehat{=}\;\; S \subseteq A \land S \cap B = \varnothing$	
*	DEF_PARTITION	$\mathrm{partition}(s, s_1, s_2, \ldots, s_n) \;\;\widehat{=}\;\; \begin{array}{ll} & s = s_1 \cup s_2 \cup \cdots \cup s_n \\ \land & s_1 \cap s_2 = \varnothing \\ \vdots & \\ \land & s_1 \cap s_n = \varnothing \\ \vdots & \\ \land & s_{n-1} \cap s_n = \varnothing \end{array}$	
	SIMP_EMPTY_PARTITION	$\mathrm{partition}(S) \;\;\widehat{=}\;\; S = \varnothing$	
	SIMP_SINGLE_PARTITION	$\mathrm{partition}(S,T) \;\;\widehat{=}\;\; S = T$	

Relation Rewrite Rules

Rules that are marked with a * in the first column are implemented in the latest version of Rodin. Rules without a * are planned to be implemented in future versions. Other conventions used in these tables are described in The_Proving_Perspective_(Rodin_User_Manual)#Rewrite_Rules.

	Name	Rule	Side Conditi
*	SIMP_DOM_SETENUM	$\mathrm{dom}(\{x \mapsto a, \ldots, y \mapsto b\}) \;\;\widehat{=}\;\; \{x, \ldots, y\}$	
*	SIMP_DOM_CONVERSE	$\mathrm{dom}(r^{-1}) \;\;\widehat{=}\;\; \mathrm{ran}(r)$	
*	SIMP_RAN_SETENUM	$\mathrm{ran}(\{x \mapsto a, \ldots, y \mapsto b\}) \;\;\widehat{=}\;\; \{a, \ldots, b\}$	
*	SIMP_RAN_CONVERSE	$\mathrm{ran}(r^{-1}) \;\;\widehat{=}\;\; \mathrm{dom}(r)$	
*	SIMP_SPECIAL_OVERL	$r \mathbin{<\!\!+} \ldots \mathbin{<\!\!+} \varnothing \mathbin{<\!\!+} \ldots \mathbin{<\!\!+} s \;\;\widehat{=}\;\; r \mathbin{<\!\!+} \ldots \mathbin{<\!\!+} s$	
*	SIMP_MULTI_OVERL	$r_1 \mathbin{<\!\!+} \cdots \mathbin{<\!\!+} r_n \;\widehat{=}\; r_1 \mathbin{<\!\!+} \cdots \mathbin{<\!\!+} r_{i-1} \mathbin{<\!\!+} r_{i+1} \mathbin{<\!\!+} \cdots \mathbin{<\!\!+} r_n$	there is a j such that $1 \leq i < j \leq n$ and syntactically equal.
*	SIMP_TYPE_OVERL_CPROD	$r \mathbin{<\!\!+} \cdots \mathbin{<\!\!+} Ty \mathbin{<\!\!+} \cdots \mathbin{<\!\!+} s \;\;\widehat{=}\;\; Ty \mathbin{<\!\!+} \cdots \mathbin{<\!\!+} s$	where Ty is a type expr
*	SIMP_SPECIAL_DOMRES_L	$\varnothing \lhd r \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_SPECIAL_DOMRES_R	$S \lhd \varnothing \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_TYPE_DOMRES	$Ty \lhd r \;\;\widehat{=}\;\; r$	where Ty is a type expr
*	SIMP_MULTI_DOMRES_DOM	$\mathrm{dom}(r) \lhd r \;\;\widehat{=}\;\; r$	
*	SIMP_MULTI_DOMRES_RAN	$\mathrm{ran}(r) \lhd r^{-1} \;\;\widehat{=}\;\; r^{-1}$	
*	SIMP_DOMRES_DOMRES_ID	$S \lhd (T \lhd \mathrm{id}) \;\;\widehat{=}\;\; (S \cap T) \lhd \mathrm{id}$	
*	SIMP_DOMRES_DOMSUB_ID	$S \lhd (T \mathbin{\lhd\!\!\!-} \mathrm{id}) \;\;\widehat{=}\;\; (S \setminus T) \lhd \mathrm{id}$	
*	SIMP_SPECIAL_RANRES_R	$r \rhd \varnothing \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_SPECIAL_RANRES_L	$\varnothing \rhd S \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_TYPE_RANRES	$r \rhd Ty \;\;\widehat{=}\;\; r$	where Ty is a type expr
*	SIMP_MULTI_RANRES_RAN	$r \rhd \mathrm{ran}(r) \;\;\widehat{=}\;\; r$	
*	SIMP_MULTI_RANRES_DOM	$r^{-1} \rhd \mathrm{dom}(r) \;\;\widehat{=}\;\; r^{-1}$	
*	SIMP_RANRES_ID	$\mathrm{id} \rhd S \;\;\widehat{=}\;\; S \lhd \mathrm{id}$	
*	SIMP_RANSUB_ID	$\mathrm{id} \mathbin{\rhd\!\!\!-} S \;\;\widehat{=}\;\; S \mathbin{\lhd\!\!\!-} \mathrm{id}$	
*	SIMP_RANRES_DOMRES_ID	$(S \lhd \mathrm{id}) \rhd T \;\;\widehat{=}\;\; (S \cap T) \lhd \mathrm{id}$	
*	SIMP_RANRES_DOMSUB_ID	$(S \mathbin{\lhd\!\!\!-} \mathrm{id}) \rhd T \;\;\widehat{=}\;\; (T \setminus S) \lhd \mathrm{id}$	
*	SIMP_SPECIAL_DOMSUB_L	$\varnothing \mathbin{\lhd\!\!\!-} r \;\;\widehat{=}\;\; r$	

*	SIMP_SPECIAL_DOMSUB_R	$S \domsub \varnothing \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_TYPE_DOMSUB	$Ty \domsub r \;\;\widehat{=}\;\; \varnothing$	where Ty is a type expı
*	SIMP_MULTI_DOMSUB_DOM	$\mathrm{dom}(r) \domsub r \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_MULTI_DOMSUB_RAN	$\mathrm{ran}(r) \domsub r^{-1} \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_DOMSUB_DOMRES_ID	$S \domsub (T \lhd \mathrm{id}) \;\;\widehat{=}\;\; (T \setminus S) \lhd \mathrm{id}$	
*	SIMP_DOMSUB_DOMSUB_ID	$S \domsub (T \domsub \mathrm{id}) \;\;\widehat{=}\;\; (S \cup T) \domsub \mathrm{id}$	
*	SIMP_SPECIAL_RANSUB_R	$r \ransub \varnothing \;\;\widehat{=}\;\; r$	
*	SIMP_SPECIAL_RANSUB_L	$\varnothing \ransub S \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_TYPE_RANSUB	$r \ransub Ty \;\;\widehat{=}\;\; \varnothing$	where Ty is a type expı
*	SIMP_MULTI_RANSUB_DOM	$r^{-1} \ransub \mathrm{dom}(r) \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_MULTI_RANSUB_RAN	$r \ransub \mathrm{ran}(r) \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_RANSUB_DOMRES_ID	$(S \lhd \mathrm{id}) \ransub T \;\;\widehat{=}\;\; (S \setminus T) \lhd \mathrm{id}$	
*	SIMP_RANSUB_DOMSUB_ID	$(S \domsub \mathrm{id}) \ransub T \;\;\widehat{=}\;\; (S \cup T) \domsub \mathrm{id}$	
*	SIMP_SPECIAL_FCOMP	$r \,;\, \ldots \,;\, \varnothing \,;\, \ldots \,;\, s \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_TYPE_FCOMP_ID	$r \,;\, \ldots \,;\, \mathrm{id} \,;\, \ldots \,;\, s \;\;\widehat{=}\;\; r \,;\, \ldots \,;\, s$	
*	SIMP_TYPE_FCOMP_R	$r \,;\, Ty \;\;\widehat{=}\;\; \mathrm{dom}(r) \times Tb$	where Ty is a type expı to $Ta \times Tb$
*	SIMP_TYPE_FCOMP_L	$Ty \,;\, r \;\;\widehat{=}\;\; Ta \times \mathrm{ran}(r)$	where Ty is a type expı to $Ta \times Tb$
*	SIMP_FCOMP_ID	$r \,;\, \ldots \,;\, S \lhd \mathrm{id} \,; T \lhd \mathrm{id} \,;\, \ldots s \;\;\widehat{=}\;\; r \,;\, \ldots \,;\, (S \cap T) \lhd \mathrm{id} \,;\, \ldots \,;\, s$	
*	SIMP_SPECIAL_BCOMP	$r \circ \ldots \circ \varnothing \circ \ldots \circ s \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_TYPE_BCOMP_ID	$r \circ \ldots \circ \mathrm{id} \circ \ldots \circ s \;\;\widehat{=}\;\; r \circ \ldots \circ s$	
*	SIMP_TYPE_BCOMP_L	$Ty \circ r \;\;\widehat{=}\;\; \mathrm{dom}(r) \times Tb$	where Ty is a type expı to $Ta \times Tb$
*	SIMP_TYPE_BCOMP_R	$r \circ Ty \;\;\widehat{=}\;\; Ta \times \mathrm{ran}(r)$	where Ty is a type expı to $Ta \times Tb$
*	SIMP_BCOMP_ID	$r \circ \ldots \circ S \lhd \mathrm{id} \circ T \lhd \mathrm{id} \circ \ldots \circ s \;\;\widehat{=}\;\; r \circ \ldots \circ (S \cap T) \lhd \mathrm{id} \circ \ldots \circ s$	
*	SIMP_SPECIAL_DPROD_R	$r \otimes \varnothing \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_SPECIAL_DPROD_L	$\varnothing \otimes r \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_DPROD_CPROD	$(S \times T) \otimes (U \times V) \;\;\widehat{=}\;\; S \cap U \times (T \times V)$	
*	SIMP_SPECIAL_PPROD_R	$r \parallel \varnothing \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_SPECIAL_PPROD_L	$\varnothing \parallel r \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_PPROD_CPROD	$(S \times T) \parallel (U \times V) \;\;\widehat{=}\;\; (S \times U) \times (T \times V)$	
*	SIMP_SPECIAL_RELIMAGE_R	$r[\varnothing] \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_SPECIAL_RELIMAGE_L	$\varnothing[S] \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_TYPE_RELIMAGE	$r[Ty] \;\;\widehat{=}\;\; \mathrm{ran}(r)$	where Ty is a type expı
*	SIMP_MULTI_RELIMAGE_DOM	$r[\mathrm{dom}(r)] \;\;\widehat{=}\;\; \mathrm{ran}(r)$	
*	SIMP_RELIMAGE_ID	$\mathrm{id}[T] \;\;\widehat{=}\;\; T$	
*	SIMP_RELIMAGE_DOMRES_ID	$(S \lhd \mathrm{id})[T] \;\;\widehat{=}\;\; S \cap T$	
*	SIMP_RELIMAGE_DOMSUB_ID	$(S \domsub \mathrm{id})[T] \;\;\widehat{=}\;\; T \setminus S$	
*	SIMP_MULTI_RELIMAGE_CPROD_SING	$(\{E\} \times S)[\{E\}] \;\;\widehat{=}\;\; S$	where E is a single ex
*	SIMP_MULTI_RELIMAGE_SING_MAPSTO	$\{E \mapsto F\}[\{E\}] \;\;\widehat{=}\;\; \{F\}$	where E is a single ex
*	SIMP_MULTI_RELIMAGE_CONVERSE_RANSUB	$(r \ransub S)^{-1}[S] \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_MULTI_RELIMAGE_CONVERSE_RANRES	$(r \rhd S)^{-1}[S] \;\;\widehat{=}\;\; r^{-1}[S]$	
*	SIMP_RELIMAGE_CONVERSE_DOMSUB	$(S \domsub r)^{-1}[T] \;\;\widehat{=}\;\; r^{-1}[T] \setminus S$	

	DERIV_RELIMAGE_RANSUB	$(r \mathbin{⩥} S)[T] \;\;\widehat{=}\;\; r[T] \setminus S$	
	DERIV_RELIMAGE_RANRES	$(r \rhd S)[T] \;\;\widehat{=}\;\; r[T] \cap S$	
*	SIMP_MULTI_RELIMAGE_DOMSUB	$(S \mathbin{⩤} r)[S] \;\;\widehat{=}\;\; \varnothing$	
	DERIV_RELIMAGE_DOMSUB	$(S \mathbin{⩤} r)[T] \;\;\widehat{=}\;\; r[T \setminus S]$	
	DERIV_RELIMAGE_DOMRES	$(S \lhd r)[T] \;\;\widehat{=}\;\; r[S \cap T]$	
*	SIMP_SPECIAL_CONVERSE	$\varnothing^{-1} \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_CONVERSE_ID	$\mathrm{id}^{-1} \;\;\widehat{=}\;\; \mathrm{id}$	
*	SIMP_CONVERSE_CPROD	$(S \times T)^{-1} \;\;\widehat{=}\;\; T \times S$	
*	SIMP_CONVERSE_SETENUM	$\{x \mapsto a, \ldots, y \mapsto b\}^{-1} \;\;\widehat{=}\;\; \{a \mapsto x, \ldots, b \mapsto y\}$	
*	SIMP_CONVERSE_COMPSET	$\{X \cdot P \mid x \mapsto y\}^{-1} \;\;\widehat{=}\;\; \{X \cdot P \mid y \mapsto x\}$	
*	SIMP_DOM_ID	$\mathrm{dom}(\mathrm{id}) \;\;\widehat{=}\;\; S$	where id has type $\mathbb{P}(S$
*	SIMP_RAN_ID	$\mathrm{ran}(\mathrm{id}) \;\;\widehat{=}\;\; S$	where id has type $\mathbb{P}(S$
*	SIMP_FCOMP_ID_L	$(S \lhd \mathrm{id}) \mathbin{;} r \;\;\widehat{=}\;\; S \lhd r$	
*	SIMP_FCOMP_ID_R	$r \mathbin{;} (S \lhd \mathrm{id}) \;\;\widehat{=}\;\; r \rhd S$	
*	SIMP_SPECIAL_REL_R	$S \leftrightarrow \varnothing \;\;\widehat{=}\;\; \{\varnothing\}$	idem for operators $\leftrightarrow\!\!\!\!\twoheadrightarrow$
*	SIMP_SPECIAL_REL_L	$\varnothing \leftrightarrow S \;\;\widehat{=}\;\; \{\varnothing\}$	idem for operators $\twoheadleftarrow\!\!\!\!\twoheadrightarrow$ $⇸$ $\rightarrow$ $⤔$ $\rightarrowtail$
*	SIMP_FUNIMAGE_PRJ1	$\mathrm{prj}_1(E \mapsto F) \;\;\widehat{=}\;\; E$	
*	SIMP_FUNIMAGE_PRJ2	$\mathrm{prj}_2(E \mapsto F) \;\;\widehat{=}\;\; F$	
*	SIMP_DOM_PRJ1	$\mathrm{dom}(\mathrm{prj}_1) \;\;\widehat{=}\;\; S \times T$	where prj_1 has type $\mathbb{P}(S \times T \times S)$
*	SIMP_DOM_PRJ2	$\mathrm{dom}(\mathrm{prj}_2) \;\;\widehat{=}\;\; S \times T$	where prj_2 has type $\mathbb{P}(S \times T \times T)$
*	SIMP_RAN_PRJ1	$\mathrm{ran}(\mathrm{prj}_1) \;\;\widehat{=}\;\; S$	where prj_1 has type $\mathbb{P}(S \times T \times S)$
*	SIMP_RAN_PRJ2	$\mathrm{ran}(\mathrm{prj}_2) \;\;\widehat{=}\;\; T$	where prj_2 has type $\mathbb{P}(S \times T \times T)$
*	SIMP_FUNIMAGE_LAMBDA	$(\lambda x \cdot P(x) \mid E(x))(y) \;\;\widehat{=}\;\; E(y)$	
*	SIMP_DOM_LAMBDA	$\mathrm{dom}(\{x \cdot P \mid E \mapsto F) \;\;\widehat{=}\;\; \{x \cdot P \mid E\}$	
*	SIMP_RAN_LAMBDA	$\mathrm{ran}(\{x \cdot P \mid E \mapsto F) \;\;\widehat{=}\;\; \{x \cdot P \mid F\}$	
*	SIMP_IN_FUNIMAGE	$E \mapsto F(E) \in F \;\;\widehat{=}\;\; \top$	
*	SIMP_IN_FUNIMAGE_CONVERSE_L	$F^{-1}(E) \mapsto E \in F \;\;\widehat{=}\;\; \top$	
*	SIMP_IN_FUNIMAGE_CONVERSE_R	$F(E) \mapsto E \in F^{-1} \;\;\widehat{=}\;\; \top$	
*	SIMP_MULTI_FUNIMAGE_SETENUM_LL	$\{A \mapsto E, \ldots, B \mapsto E\}(x) \;\;\widehat{=}\;\; E$	
*	SIMP_MULTI_FUNIMAGE_SETENUM_LR	$\{E, \ldots, x \mapsto y, \ldots, F\}(x) \;\;\widehat{=}\;\; y$	
*	SIMP_MULTI_FUNIMAGE_OVERL_SETENUM	$(r \mathbin{\lhd\!\!\!-} \ldots \mathbin{\lhd\!\!\!-} \{E, \ldots, x \mapsto y, \ldots, F\})(x) \;\;\widehat{=}\;\; y$	
*	SIMP_MULTI_FUNIMAGE_BUNION_SETENUM	$(r \cup \ldots \cup \{E, \ldots, x \mapsto y, \ldots, F\})(x) \;\;\widehat{=}\;\; y$	
*	SIMP_FUNIMAGE_CPROD	$(S \times \{F\})(x) \;\;\widehat{=}\;\; F$	
*	SIMP_FUNIMAGE_ID	$\mathrm{id}(x) \;\;\widehat{=}\;\; x$	
*	SIMP_FUNIMAGE_FUNIMAGE_CONVERSE	$f(f^{-1}(E)) \;\;\widehat{=}\;\; E$	
*	SIMP_FUNIMAGE_CONVERSE_FUNIMAGE	$f^{-1}(f(E)) \;\;\widehat{=}\;\; E$	
*	SIMP_FUNIMAGE_FUNIMAGE_CONVERSE_SETENUM	$\{x \mapsto a, \ldots, y \mapsto b\}(\{a \mapsto x, \ldots, b \mapsto y\}(E)) \;\;\widehat{=}\;\; E$	
*	SIMP_FUNIMAGE_DOMRES	$(E \lhd F)(G) \;\;\widehat{=}\;\; F(G)$	with hypothesis $F \in A$ where op is one of $⇸$, $⤀$, $\twoheadrightarrow$, $⤖$.

*	SIMP_FUNIMAGE_DOMSUB	$(E ⩤ F)(G) \;\;\widehat{=}\;\; F(G)$	with hypothesis $F \in A$ where op is one of ⇸, ⤀, ↠, ⤖.
*	SIMP_FUNIMAGE_RANRES	$(F \rhd E)(G) \;\;\widehat{=}\;\; F(G)$	with hypothesis $F \in A$ where op is one of ⇸, ⤀, ↠, ⤖.
*	SIMP_FUNIMAGE_RANSUB	$(F ⩥ E)(G) \;\;\widehat{=}\;\; F(G)$	with hypothesis $F \in A$ where op is one of ⇸, ⤀, ↠, ⤖.
*	SIMP_FUNIMAGE_SETMINUS	$(F \setminus E)(G) \;\;\widehat{=}\;\; F(G)$	with hypothesis $F \in A$ where op is one of ⇸, ⤀, ↠, ⤖.
	DEF_BCOMP	$r \circ \ldots \circ s \;\;\widehat{=}\;\; s \,;\ldots;\, r$	
	DERIV_ID_SING	$\{E\} \lhd \mathrm{id} \;\;\widehat{=}\;\; \{E \mapsto E\}$	where E is a single ex
*	SIMP_SPECIAL_DOM	$\mathrm{dom}(\varnothing) \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_SPECIAL_RAN	$\mathrm{ran}(\varnothing) \;\;\widehat{=}\;\; \varnothing$	
*	SIMP_CONVERSE_CONVERSE	$r^{-1-1} \;\;\widehat{=}\;\; r$	
*	DERIV_RELIMAGE_FCOMP	$(p\,;q)[s] \;\;\widehat{=}\;\; q[p[s]]$	
*	DERIV_FCOMP_DOMRES	$(s \lhd p)\,;q \;\;\widehat{=}\;\; s \lhd (p\,;q)$	
*	DERIV_FCOMP_DOMSUB	$(s ⩤ p)\,;q \;\;\widehat{=}\;\; s ⩤ (p\,;q)$	
*	DERIV_FCOMP_RANRES	$p\,;(q \rhd s) \;\;\widehat{=}\;\; (p\,;q) \rhd s$	
*	DERIV_FCOMP_RANSUB	$p\,;(q ⩥ s) \;\;\widehat{=}\;\; (p\,;q) ⩥ s$	
	DERIV_FCOMP_SING	$\{E \mapsto F\}\,;\{F \mapsto G\} \;\;\widehat{=}\;\; \{E \mapsto G\}$	
*	SIMP_SPECIAL_EQUAL_RELDOMRAN	$\varnothing$ ⟷↠ $\varnothing \;\;\widehat{=}\;\; \{\varnothing\}$	idem for operators ↠ ⤖
*	SIMP_TYPE_DOM	$\mathrm{dom}(Ty) \;\;\widehat{=}\;\; Ta$	where Ty is a type expr to $Ta \times Tb$
*	SIMP_TYPE_RAN	$\mathrm{ran}(Ty) \;\;\widehat{=}\;\; Tb$	where Ty is a type expr to $Ta \times Tb$
*	SIMP_MULTI_DOM_CPROD	$\mathrm{dom}(E \times E) \;\;\widehat{=}\;\; E$	
*	SIMP_MULTI_RAN_CPROD	$\mathrm{ran}(E \times E) \;\;\widehat{=}\;\; E$	
*	SIMP_MULTI_DOM_DOMRES	$\mathrm{dom}(A \lhd f) \;\;\widehat{=}\;\; \mathrm{dom}(f) \cap A$	
*	SIMP_MULTI_DOM_DOMSUB	$\mathrm{dom}(A ⩤ f) \;\;\widehat{=}\;\; \mathrm{dom}(f) \setminus A$	
*	SIMP_MULTI_RAN_RANRES	$\mathrm{ran}(f \rhd A) \;\;\widehat{=}\;\; \mathrm{ran}(f) \cap A$	
*	SIMP_MULTI_RAN_RANSUB	$\mathrm{ran}(f ⩥ A) \;\;\widehat{=}\;\; \mathrm{ran}(f) \setminus A$	
*	DEF_IN_DOM	$E \in \mathrm{dom}(r) \;\;\widehat{=}\;\; \exists y \cdot E \mapsto y \in r$	
*	DEF_IN_RAN	$F \in \mathrm{ran}(r) \;\;\widehat{=}\;\; \exists x \cdot x \mapsto F \in r$	
*	DEF_IN_CONVERSE	$E \mapsto F \in r^{-1} \;\;\widehat{=}\;\; F \mapsto E \in r$	
*	DEF_IN_DOMRES	$E \mapsto F \in S \lhd r \;\;\widehat{=}\;\; E \in S \land E \mapsto F \in r$	
*	DEF_IN_RANRES	$E \mapsto F \in r \rhd T \;\;\widehat{=}\;\; E \mapsto F \in r \land F \in T$	
*	DEF_IN_DOMSUB	$E \mapsto F \in S ⩤ r \;\;\widehat{=}\;\; E \notin S \land E \mapsto F \in r$	
*	DEF_IN_RANSUB	$E \mapsto F \in r \rhd T \;\;\widehat{=}\;\; E \mapsto F \in r \land F \notin T$	
*	DEF_IN_RELIMAGE	$F \in r[w] \;\;\widehat{=}\;\; \exists x \cdot x \in w \land x \mapsto F \in r$	
*	DEF_IN_FCOMP	$E \mapsto F \in (p\,;q) \;\;\widehat{=}\;\; \exists x \cdot E \mapsto x \in p \land x \mapsto F \in q$	
*	DEF_OVERL	$p \mathbin{<\!\!+} q \;\;\widehat{=}\;\; (\mathrm{dom}(q) ⩤ p) \cup q$	
*	DEF_IN_ID	$E \mapsto F \in \mathrm{id} \;\;\widehat{=}\;\; E = F$	
*	DEF_IN_DPROD	$E \mapsto (F \mapsto G) \in p \otimes q \;\;\widehat{=}\;\; E \mapsto F \in p \land E \mapsto G \in q$	
*	DEF_IN_PPROD	$(E \mapsto G) \mapsto (F \mapsto H) \in p \parallel q \;\;\widehat{=}\;\; E \mapsto F \in p \land G \mapsto H \in q$	
*	DEF_IN_REL	$r \in S \leftrightarrow T \;\;\widehat{=}\;\; r \subseteq S \times T$	

*	DEF_IN_RELDOM	$r \in S \mathrel{\leftarrow\!\!\!\!\leftrightarrow} T \mathrel{\widehat{=}} r \in S \leftrightarrow T \land \mathrm{dom}(r) = S$	
*	DEF_IN_RELRAN	$r \in S \mathrel{\leftrightarrow\!\!\!\!\rightarrow} T \mathrel{\widehat{=}} r \in S \leftrightarrow T \land \mathrm{ran}(r) = T$	
*	DEF_IN_RELDOMRAN	$r \in S \mathrel{\leftarrow\!\!\!\!\leftrightarrow\!\!\!\!\rightarrow} T \mathrel{\widehat{=}} r \in S \leftrightarrow T \land \mathrm{dom}(r) = S \land \mathrm{ran}(r) = T$	
*	DEF_IN_FCT	$f \in S \mathrel{+\!\!\!\!\to} T \mathrel{\widehat{=}} f \in S \leftrightarrow T \land (\forall x, y, z \cdot x \mapsto y \in f \land x \mapsto z \in f \Rightarrow y = z)$	
*	DEF_IN_TFCT	$f \in S \rightarrow T \mathrel{\widehat{=}} f \in S \mathrel{+\!\!\!\!\to} T \land \mathrm{dom}(f) = S$	
*	DEF_IN_INJ	$f \in S \mathrel{\rightarrowtail\!\!\!\!\!\!+} T \mathrel{\widehat{=}} f \in S \mathrel{+\!\!\!\!\to} T \land f^{-1} \in T \mathrel{+\!\!\!\!\to} S$	
*	DEF_IN_TINJ	$f \in S \rightarrowtail T \mathrel{\widehat{=}} f \in S \mathrel{\rightarrowtail\!\!\!\!\!\!+} T \land \mathrm{dom}(f) = S$	
*	DEF_IN_SURJ	$f \in S \mathrel{+\!\!\!\!\twoheadrightarrow} T \mathrel{\widehat{=}} f \in S \mathrel{+\!\!\!\!\to} T \land \mathrm{ran}(f) = T$	
*	DEF_IN_TSURJ	$f \in S \twoheadrightarrow T \mathrel{\widehat{=}} f \in S \mathrel{+\!\!\!\!\twoheadrightarrow} T \land \mathrm{dom}(f) = S$	
*	DEF_IN_BIJ	$f \in S \mathrel{\rightarrowtail\!\!\!\!\twoheadrightarrow} T \mathrel{\widehat{=}} f \in S \rightarrowtail T \land \mathrm{ran}(f) = T$	
	DISTRI_BCOMP_BUNION	$r \circ (s \cup t) \mathrel{\widehat{=}} (r \circ s) \cup (r \circ t)$	
*	DISTRI_FCOMP_BUNION_R	$p \mathbin{;} (q \cup r) \mathrel{\widehat{=}} (p \mathbin{;} q) \cup (p \mathbin{;} r)$	
*	DISTRI_FCOMP_BUNION_L	$(q \cup r) \mathbin{;} p \mathrel{\widehat{=}} (q \mathbin{;} p) \cup (r \mathbin{;} p)$	
	DISTRI_DPROD_BUNION	$r \otimes (s \cup t) \mathrel{\widehat{=}} (r \otimes s) \cup (r \otimes t)$	
	DISTRI_DPROD_BINTER	$r \otimes (s \cap t) \mathrel{\widehat{=}} (r \otimes s) \cap (r \otimes t)$	
	DISTRI_DPROD_SETMINUS	$r \otimes (s \setminus t) \mathrel{\widehat{=}} (r \otimes s) \setminus (r \otimes t)$	
	DISTRI_DPROD_OVERL	$r \otimes (s \mathbin{<\!\!\!+} t) \mathrel{\widehat{=}} (r \otimes s) \mathbin{<\!\!\!+} (r \otimes t)$	
	DISTRI_PPROD_BUNION	$r \parallel (s \cup t) \mathrel{\widehat{=}} (r \parallel s) \cup (r \parallel t)$	
	DISTRI_PPROD_BINTER	$r \parallel (s \cap t) \mathrel{\widehat{=}} (r \parallel s) \cap (r \parallel t)$	
	DISTRI_PPROD_SETMINUS	$r \parallel (s \setminus t) \mathrel{\widehat{=}} (r \parallel s) \setminus (r \parallel t)$	
	DISTRI_PPROD_OVERL	$r \parallel (s \mathbin{<\!\!\!+} t) \mathrel{\widehat{=}} (r \parallel s) \mathbin{<\!\!\!+} (r \parallel t)$	
	DISTRI_OVERL_BUNION_L	$(p \cup q) \mathbin{<\!\!\!+} r \mathrel{\widehat{=}} (p \mathbin{<\!\!\!+} r) \cup (q \mathbin{<\!\!\!+} r)$	
	DISTRI_OVERL_BINTER_L	$(p \cap q) \mathbin{<\!\!\!+} r \mathrel{\widehat{=}} (p \mathbin{<\!\!\!+} r) \cap (q \mathbin{<\!\!\!+} r)$	
*	DISTRI_DOMRES_BUNION_R	$s \lhd (p \cup q) \mathrel{\widehat{=}} (s \lhd p) \cup (s \lhd q)$	
*	DISTRI_DOMRES_BUNION_L	$(s \cup t) \lhd r \mathrel{\widehat{=}} (s \lhd r) \cup (t \lhd r)$	
*	DISTRI_DOMRES_BINTER_R	$s \lhd (p \cap q) \mathrel{\widehat{=}} (s \lhd p) \cap (s \lhd q)$	
*	DISTRI_DOMRES_BINTER_L	$(s \cap t) \lhd r \mathrel{\widehat{=}} (s \lhd r) \cap (t \lhd r)$	
	DISTRI_DOMRES_SETMINUS_R	$s \lhd (p \setminus q) \mathrel{\widehat{=}} (s \lhd p) \setminus (s \lhd q)$	
	DISTRI_DOMRES_SETMINUS_L	$(s \setminus t) \lhd r \mathrel{\widehat{=}} (s \lhd r) \setminus (t \lhd r)$	
	DISTRI_DOMRES_DPROD	$s \lhd (p \otimes q) \mathrel{\widehat{=}} (s \lhd p) \otimes (s \lhd q)$	
	DISTRI_DOMRES_OVERL	$s \lhd (r \mathbin{<\!\!\!+} q) \mathrel{\widehat{=}} (s \lhd r) \mathbin{<\!\!\!+} (s \lhd q)$	
*	DISTRI_DOMSUB_BUNION_R	$s \mathbin{\lhd\!\!\!-} (p \cup q) \mathrel{\widehat{=}} (s \mathbin{\lhd\!\!\!-} p) \cup (s \mathbin{\lhd\!\!\!-} q)$	
*	DISTRI_DOMSUB_BUNION_L	$(s \cup t) \mathbin{\lhd\!\!\!-} r \mathrel{\widehat{=}} (s \mathbin{\lhd\!\!\!-} r) \cap (t \mathbin{\lhd\!\!\!-} r)$	
*	DISTRI_DOMSUB_BINTER_R	$s \mathbin{\lhd\!\!\!-} (p \cap q) \mathrel{\widehat{=}} (s \mathbin{\lhd\!\!\!-} p) \cap (s \mathbin{\lhd\!\!\!-} q)$	
*	DISTRI_DOMSUB_BINTER_L	$(s \cap t) \mathbin{\lhd\!\!\!-} r \mathrel{\widehat{=}} (s \mathbin{\lhd\!\!\!-} r) \cup (t \mathbin{\lhd\!\!\!-} r)$	
	DISTRI_DOMSUB_DPROD	$A \mathbin{\lhd\!\!\!-} (r \otimes s) \mathrel{\widehat{=}} (A \mathbin{\lhd\!\!\!-} r) \otimes (A \mathbin{\lhd\!\!\!-} s)$	
	DISTRI_DOMSUB_OVERL	$A \mathbin{\lhd\!\!\!-} (r \mathbin{<\!\!\!+} s) \mathrel{\widehat{=}} (A \mathbin{\lhd\!\!\!-} r) \mathbin{<\!\!\!+} (A \mathbin{\lhd\!\!\!-} s)$	
*	DISTRI_RANRES_BUNION_R	$r \rhd (s \cup t) \mathrel{\widehat{=}} (r \rhd s) \cup (r \rhd t)$	
*	DISTRI_RANRES_BUNION_L	$(p \cup q) \rhd s \mathrel{\widehat{=}} (p \rhd s) \cup (q \rhd s)$	
*	DISTRI_RANRES_BINTER_R	$r \rhd (s \cap t) \mathrel{\widehat{=}} (r \rhd s) \cap (r \rhd t)$	
*	DISTRI_RANRES_BINTER_L	$(p \cap q) \rhd s \mathrel{\widehat{=}} (p \rhd s) \cap (q \rhd s)$	

	DISTRI_RANRES_SETMINUS_R	$r \rhd (s \setminus t) \ \widehat{=}\ (r \rhd s) \setminus (r \rhd t)$	
	DISTRI_RANRES_SETMINUS_L	$(p \setminus q) \rhd s \ \widehat{=}\ (p \rhd s) \setminus (q \rhd s)$	
*	DISTRI_RANSUB_BUNION_R	$r ⩥ (s \cup t) \ \widehat{=}\ (r ⩥ s) \cap (r ⩥ t)$	
*	DISTRI_RANSUB_BUNION_L	$(p \cup q) ⩥ s \ \widehat{=}\ (p ⩥ s) \cup (q ⩥ s)$	
*	DISTRI_RANSUB_BINTER_R	$r ⩥ (s \cap t) \ \widehat{=}\ (r ⩥ s) \cup (r ⩥ t)$	
*	DISTRI_RANSUB_BINTER_L	$(p \cap q) ⩥ s \ \widehat{=}\ (p ⩥ s) \cap (q ⩥ s)$	
*	DISTRI_CONVERSE_BUNION	$(p \cup q)^{-1} \ \widehat{=}\ p^{-1} \cup q^{-1}$	
	DISTRI_CONVERSE_BINTER	$(p \cap q)^{-1} \ \widehat{=}\ p^{-1} \cap q^{-1}$	
	DISTRI_CONVERSE_SETMINUS	$(r \setminus s)^{-1} \ \widehat{=}\ r^{-1} \setminus s^{-1}$	
	DISTRI_CONVERSE_BCOMP	$(r \circ s)^{-1} \ \widehat{=}\ (s^{-1} \circ r^{-1})$	
	DISTRI_CONVERSE_FCOMP	$(p \,;\, q)^{-1} \ \widehat{=}\ (q^{-1} \,;\, p^{-1})$	
	DISTRI_CONVERSE_PPROD	$(r \parallel s)^{-1} \ \widehat{=}\ r^{-1} \parallel s^{-1}$	
	DISTRI_CONVERSE_DOMRES	$(s \lhd r)^{-1} \ \widehat{=}\ r^{-1} \rhd s$	
	DISTRI_CONVERSE_DOMSUB	$(s ⩤ r)^{-1} \ \widehat{=}\ r^{-1} ⩥ s$	
	DISTRI_CONVERSE_RANRES	$(r \rhd s)^{-1} \ \widehat{=}\ s \lhd r^{-1}$	
	DISTRI_CONVERSE_RANSUB	$(r ⩥ s)^{-1} \ \widehat{=}\ s ⩤ r^{-1}$	
*	DISTRI_DOM_BUNION	$\mathrm{dom}(r \cup s) \ \widehat{=}\ \mathrm{dom}(r) \cup \mathrm{dom}(s)$	
*	DISTRI_RAN_BUNION	$\mathrm{ran}(r \cup s) \ \widehat{=}\ \mathrm{ran}(r) \cup \mathrm{ran}(s)$	
*	DISTRI_RELIMAGE_BUNION_R	$r[S \cup T] \ \widehat{=}\ r[S] \cup r[T]$	
*	DISTRI_RELIMAGE_BUNION_L	$(p \cup q)[S] \ \widehat{=}\ p[S] \cup q[S]$	
*	DERIV_DOM_TOTALREL	$\mathrm{dom}(r) \ \widehat{=}\ E$	with hypothesis $r \in E$ where op is one of $\mathrel{\leftarrow\!\!\!\!\leftrightarrow}$, $\mathrel{\leftarrow\!\!\!\!\leftrightarrow\!\!\!\!\rightarrow}$, $\rightarrow$, $\rightarrowtail$, $\twoheadrightarrow$,
	DERIV_RAN_SURJREL	$\mathrm{ran}(r) \ \widehat{=}\ F$	with hypothesis $r \in E$ where op is one of $\mathrel{\leftrightarrow\!\!\!\!\rightarrow}$, $\mathrel{\leftarrow\!\!\!\!\leftrightarrow\!\!\!\!\rightarrow}$, ⤀, $\twoheadrightarrow$, ⤖
*	DERIV_PRJ1_SURJ	$\mathrm{prj}_1 \in Ty_1 \ op \ Ty_2 \ \widehat{=}\ \top$	where Ty_1 and Ty_2 are op is one of $\leftrightarrow$, $\mathrel{\leftarrow\!\!\!\!\leftrightarrow}$, $\mathrel{\leftrightarrow\!\!\!\!\rightarrow}$, $\mathrel{\leftarrow\!\!\!\!\leftrightarrow\!\!\!\!\rightarrow}$, ⇸,
*	DERIV_PRJ2_SURJ	$\mathrm{prj}_2 \in Ty_1 \ op \ Ty_2 \ \widehat{=}\ \top$	where Ty_1 and Ty_2 are op is one of $\leftrightarrow$, $\mathrel{\leftarrow\!\!\!\!\leftrightarrow}$, $\mathrel{\leftrightarrow\!\!\!\!\rightarrow}$, $\mathrel{\leftarrow\!\!\!\!\leftrightarrow\!\!\!\!\rightarrow}$, ⇸,
*	DERIV_ID_BIJ	$\mathrm{id} \in Ty \ op \ Ty \ \widehat{=}\ \top$	where Ty is a type and arrow
*	SIMP_MAPSTO_PRJ1_PRJ2	$\mathrm{prj}_1(E) \mapsto \mathrm{prj}_2(E) \ \widehat{=}\ E$	
	DERIV_EXPAND_PRJS	$E \ \widehat{=}\ \mathrm{prj}_1(E) \mapsto \mathrm{prj}_2(E)$	
*	SIMP_DOM_SUCC	$\mathrm{dom}(\mathrm{succ}) \ \widehat{=}\ \mathbb{Z}$	
*	SIMP_RAN_SUCC	$\mathrm{ran}(\mathrm{succ}) \ \widehat{=}\ \mathbb{Z}$	
*	DERIV_MULTI_IN_BUNION	$E \in A \cup \cdots \cup \{\cdots, E, \cdots\} \cup \cdots \cup B \ \widehat{=}\ \top$	
*	DERIV_MULTI_IN_SETMINUS	$E \in S \setminus \{\cdots, E, \cdots\} \ \widehat{=}\ \bot$	
*	DEF_PRED	$\mathrm{pred} \ \widehat{=}\ \mathrm{succ}^{-1}$	

Empty Set Rewrite Rules

Rules that are marked with a * in the first column are implemented in the latest version of Rodin. Rules without a * are planned to be implemented in future versions. Other conventions used in these tables are described in The_Proving_Perspective_(Rodin_User_Manual)#Rewrite_Rules.

All rewrite rules that match the pattern $\mathbf{P} = \emptyset$ are also applicable to predicates of the form $\mathbf{P} \subseteq \emptyset$ and $\emptyset = \mathbf{P}$, as these predicates are equivalent. All rewrite rules that match the pattern $\mathbf{P} = Ty$ are also applicable to predicates of the form $Ty \subseteq \mathbf{P}$ and $Ty = \mathbf{P}$, as these predicates are equivalent.

	Name	Rule	Side Conditio
*	DEF_SPECIAL_NOT_EQUAL	$\neg\, S = \varnothing \;\;\widehat{=}\;\; \exists x \cdot x \in S$	where x is not free in
*	SIMP_SETENUM_EQUAL_EMPTY	$\{A, \ldots, B\} = \varnothing \;\;\widehat{=}\;\; \bot$	
*	SIMP_SPECIAL_EQUAL_COMPSET	$\{x \cdot P(x) \mid E\} = \varnothing \;\;\widehat{=}\;\; \forall x \cdot \neg\, P(x)$	
*	SIMP_BINTER_EQUAL_TYPE	$A \cap \cdots \cap B = Ty \;\;\widehat{=}\;\; A = Ty \land \cdots \land B = Ty$	where Ty is a type exp
*	SIMP_BINTER_SING_EQUAL_EMPTY	$A \cap \cdots \cap \{a\} \cap \cdots \cap B = \varnothing \;\;\widehat{=}\;\; \neg\, a \in A \cap \cdots \cap B$	
*	SIMP_BINTER_SETMINUS_EQUAL_EMPTY	$A \cap \cdots \cap (B \setminus C) \cap \cdots \cap D = \varnothing \;\;\widehat{=}\;\; (A \cap \cdots \cap B \cap \cdots \cap D) \setminus C = \varnothing$	
*	SIMP_BUNION_EQUAL_EMPTY	$A \cup \cdots \cup B = \varnothing \;\;\widehat{=}\;\; A = \varnothing \land \cdots \land B = \varnothing$	
*	SIMP_SETMINUS_EQUAL_EMPTY	$A \setminus B = \varnothing \;\;\widehat{=}\;\; A \subseteq B$	
*	SIMP_SETMINUS_EQUAL_TYPE	$A \setminus B = Ty \;\;\widehat{=}\;\; A = Ty \land B = \varnothing$	where Ty is a type exp
*	SIMP_POW_EQUAL_EMPTY	$\mathbb{P}(S) = \varnothing \;\;\widehat{=}\;\; \bot$	
*	SIMP_POW1_EQUAL_EMPTY	$\mathbb{P}_1(S) = \varnothing \;\;\widehat{=}\;\; S = \varnothing$	
*	SIMP_KINTER_EQUAL_TYPE	$\mathrm{inter}(S) = Ty \;\;\widehat{=}\;\; S = \{Ty\}$	where Ty is a type exp
*	SIMP_KUNION_EQUAL_EMPTY	$\mathrm{union}(S) = \varnothing \;\;\widehat{=}\;\; S \subseteq \{\varnothing\}$	
*	SIMP_QINTER_EQUAL_TYPE	$(\bigcap x \cdot P(x) \mid E(x)) = Ty \;\;\widehat{=}\;\; \forall x \cdot P(x) \Rightarrow E(x) = Ty$	where Ty is a type exp
*	SIMP_QUNION_EQUAL_EMPTY	$(\bigcup x \cdot P(x) \mid E(x)) = \varnothing \;\;\widehat{=}\;\; \forall x \cdot P(x) \Rightarrow E(x) = \varnothing$	
*	SIMP_NATURAL_EQUAL_EMPTY	$\mathbb{N} = \varnothing \;\;\widehat{=}\;\; \bot$	
*	SIMP_NATURAL1_EQUAL_EMPTY	$\mathbb{N}_1 = \varnothing \;\;\widehat{=}\;\; \bot$	
*	SIMP_TYPE_EQUAL_EMPTY	$Ty = \varnothing \;\;\widehat{=}\;\; \bot$	where Ty is a type exp
*	SIMP_CPROD_EQUAL_EMPTY	$S \times T = \varnothing \;\;\widehat{=}\;\; S = \varnothing \lor T = \varnothing$	
*	SIMP_CPROD_EQUAL_TYPE	$S \times T = Ty \;\;\widehat{=}\;\; S = Ta \land T = Tb$	where Ty is a type exp equal to $Ta \times Tb$
*	SIMP_UPTO_EQUAL_EMPTY	$i \mathbin{..} j = \varnothing \;\;\widehat{=}\;\; i > j$	
*	SIMP_UPTO_EQUAL_INTEGER	$i \mathbin{..} j = \mathbb{Z} \;\;\widehat{=}\;\; \bot$	
*	SIMP_UPTO_EQUAL_NATURAL	$i \mathbin{..} j = \mathbb{N} \;\;\widehat{=}\;\; \bot$	
*	SIMP_UPTO_EQUAL_NATURAL1	$i \mathbin{..} j = \mathbb{N}_1 \;\;\widehat{=}\;\; \bot$	
*	SIMP_SPECIAL_EQUAL_REL	$A \leftrightarrow B = \varnothing \;\;\widehat{=}\;\; \bot$	idem for operators $\mathrel{+\!\!\!\!\rightarrow}$,
	SIMP_TYPE_EQUAL_REL	$A \leftrightarrow B = Ty \;\;\widehat{=}\;\; A = Ta \land B = Tb$	where Ty is a type exp equal to $Ta \times Tb$
*	SIMP_SPECIAL_EQUAL_RELDOM	$A \mathrel{\leftarrow\!\!\!\!\leftrightarrow} B = \varnothing \;\;\widehat{=}\;\; \neg\, A = \varnothing \land B = \varnothing$	idem for operator $\rightarrow$
	SIMP_TYPE_EQUAL_RELDOMRAN	$A \mathrel{\leftarrow\!\!\!\!\leftrightarrow} B = Ty \;\;\widehat{=}\;\; \bot$	where Ty is a type exp idem for operator $\mathrel{\leftrightarrow\!\!\!\!\rightarrow}$, $\mathrel{\leftarrow\!\!\!\!\leftrightarrow\!\!\!\!\rightarrow}$, $\rightarrow$, $\rightarrowtail$, $\mathrel{+\!\!\!\!\twoheadrightarrow}$
*	SIMP_SREL_EQUAL_EMPTY	$A \mathrel{\leftrightarrow\!\!\!\!\rightarrow} B = \varnothing \;\;\widehat{=}\;\; A = \varnothing \land \neg\, B = \varnothing$	
*	SIMP_STREL_EQUAL_EMPTY	$A \mathrel{\leftarrow\!\!\!\!\leftrightarrow\!\!\!\!\rightarrow} B = \varnothing \;\;\widehat{=}\;\; (A = \varnothing \Leftrightarrow \neg\, B = \varnothing)$	
*	SIMP_DOM_EQUAL_EMPTY	$\mathrm{dom}(r) = \varnothing \;\;\widehat{=}\;\; r = \varnothing$	
*	SIMP_RAN_EQUAL_EMPTY	$\mathrm{ran}(r) = \varnothing \;\;\widehat{=}\;\; r = \varnothing$	
*	SIMP_FCOMP_EQUAL_EMPTY	$p \mathbin{;} q = \varnothing \;\;\widehat{=}\;\; \mathrm{ran}(p) \cap \mathrm{dom}(q) = \varnothing$	
*	SIMP_BCOMP_EQUAL_EMPTY	$p \circ q = \varnothing \;\;\widehat{=}\;\; \mathrm{ran}(q) \cap \mathrm{dom}(p) = \varnothing$	
*	SIMP_DOMRES_EQUAL_EMPTY	$S \lhd r = \varnothing \;\;\widehat{=}\;\; \mathrm{dom}(r) \cap S = \varnothing$	
*	SIMP_DOMRES_EQUAL_TYPE	$S \lhd r = Ty \;\;\widehat{=}\;\; S = Ta \land r = Ty$	where Ty is a type exp equal to $Ta \times Tb$
*	SIMP_DOMSUB_EQUAL_EMPTY	$S \mathbin{\triangleleft\!\!\!-} r = \varnothing \;\;\widehat{=}\;\; \mathrm{dom}(r) \subseteq S$	
*	SIMP_DOMSUB_EQUAL_TYPE	$S \mathbin{\triangleleft\!\!\!-} r = Ty \;\;\widehat{=}\;\; S = \varnothing \land r = Ty$	where Ty is a type exp
*	SIMP_RANRES_EQUAL_EMPTY	$r \rhd S = \varnothing \;\;\widehat{=}\;\; \mathrm{ran}(r) \cap S = \varnothing$	

*	SIMP_RANRES_EQUAL_TYPE	$r \rhd S = Ty \;\; \widehat{=} \;\; S = Tb \land r = Ty$	where Ty is a type exp equal to $Ta \times Tb$
*	SIMP_RANSUB_EQUAL_EMPTY	$r \mathbin{⩥} S = \varnothing \;\; \widehat{=} \;\; \mathrm{ran}(r) \subseteq S$	
*	SIMP_RANSUB_EQUAL_TYPE	$r \mathbin{⩥} S = Ty \;\; \widehat{=} \;\; S = \varnothing \land r = Ty$	where Ty is a type exp
*	SIMP_CONVERSE_EQUAL_EMPTY	$r^{-1} = \varnothing \;\; \widehat{=} \;\; r = \varnothing$	
*	SIMP_CONVERSE_EQUAL_TYPE	$r^{-1} = Ty \;\; \widehat{=} \;\; r = Ty^{-1}$	where Ty is a type exp
*	SIMP_RELIMAGE_EQUAL_EMPTY	$r[S] = \varnothing \;\; \widehat{=} \;\; S \lhd r = \varnothing$	
*	SIMP_OVERL_EQUAL_EMPTY	$r \mathbin{⩤} \cdots \mathbin{⩤} s = \varnothing \;\; \widehat{=} \;\; r = \varnothing \land \cdots \land s = \varnothing$	
*	SIMP_DPROD_EQUAL_EMPTY	$p \otimes q = \varnothing \;\; \widehat{=} \;\; \mathrm{dom}(p) \cap \mathrm{dom}(q) = \varnothing$	
*	SIMP_DPROD_EQUAL_TYPE	$p \otimes q = Ty \;\; \widehat{=} \;\; p = Ta \times Tb \land q = Ta \times Tc$	where Ty is a type exp equal to $Ta \times (Tb \times$
*	SIMP_PPROD_EQUAL_EMPTY	$p \parallel q = \varnothing \;\; \widehat{=} \;\; p = \varnothing \lor q = \varnothing$	
*	SIMP_PPROD_EQUAL_TYPE	$p \parallel q = Ty \;\; \widehat{=} \;\; p = Ta \times Tc \land q = Tb \times Td$	where Ty is a type exp equal to $(Ta \times Tb) \times (Tc$
*	SIMP_ID_EQUAL_EMPTY	$\mathrm{id} = \varnothing \;\; \widehat{=} \;\; \bot$	
*	SIMP_PRJ1_EQUAL_EMPTY	$\mathrm{prj}_1 = \varnothing \;\; \widehat{=} \;\; \bot$	
*	SIMP_PRJ2_EQUAL_EMPTY	$\mathrm{prj}_2 = \varnothing \;\; \widehat{=} \;\; \bot$	

Arithmetic Rewrite Rules

Rules that are marked with a * in the first column are implemented in the latest version of Rodin. Rules without a * are planned to be implemented in future versions. Other conventions used in these tables are described in The_Proving_Perspective_(Rodin_User_Manual)#Rewrite_Rules.

	Name	Rule	Side Conditi
*	SIMP_SPECIAL_MOD_0	$0 \bmod E \;\; \widehat{=} \;\; 0$	
*	SIMP_SPECIAL_MOD_1	$E \bmod 1 \;\; \widehat{=} \;\; 0$	
*	SIMP_MIN_SING	$\min(\{E\}) \;\; \widehat{=} \;\; E$	where E is a single expressio
*	SIMP_MAX_SING	$\max(\{E\}) \;\; \widehat{=} \;\; E$	where E is a single expressio
*	SIMP_MIN_NATURAL	$\min(\mathbb{N}) \;\; \widehat{=} \;\; 0$	
*	SIMP_MIN_NATURAL1	$\min(\mathbb{N}_1) \;\; \widehat{=} \;\; 1$	
*	SIMP_MIN_BUNION_SING	$\min(S \cup \ldots \cup \{\min(T)\} \cup \ldots \cup U)$ $\widehat{=} \;\; \min(S \cup \ldots \cup T \cup \ldots \cup U)$	
*	SIMP_MAX_BUNION_SING	$\max(S \cup \ldots \cup \{\max(T)\} \cup \ldots \cup U)$ $\widehat{=} \;\; \max(S \cup \ldots \cup T \cup \ldots \cup U)$	
*	SIMP_MIN_UPTO	$\min(E \mathbin{..} F) \;\; \widehat{=} \;\; E$	
*	SIMP_MAX_UPTO	$\max(E \mathbin{..} F) \;\; \widehat{=} \;\; F$	
*	SIMP_LIT_MIN	$\min(\{E, \ldots, i, \ldots, j, \ldots, H\}) \;\; \widehat{=} \;\; \min(\{E, \ldots, i, \ldots, H\})$	where i and j a literals and $i \leq$
*	SIMP_LIT_MAX	$\max(\{E, \ldots, i, \ldots, j, \ldots, H\}) \;\; \widehat{=} \;\; \max(\{E, \ldots, i, \ldots, H\})$	where i and j a literals and $i \geq$
*	SIMP_SPECIAL_CARD	$\mathrm{card}(\varnothing) \;\; \widehat{=} \;\; 0$	
*	SIMP_CARD_SING	$\mathrm{card}(\{E\}) \;\; \widehat{=} \;\; 1$	where E is a single expressio
*	SIMP_SPECIAL_EQUAL_CARD	$\mathrm{card}(S) = 0 \;\; \widehat{=} \;\; S = \varnothing$	
*	SIMP_CARD_POW	$\mathrm{card}(\mathbb{P}(S)) \;\; \widehat{=} \;\; 2 \hat{} \mathrm{card}(S)$	
*	SIMP_CARD_BUNION	$\mathrm{card}(S \cup T) \;\; \widehat{=} \;\; \mathrm{card}(S) + \mathrm{card}(T) - \mathrm{card}(S \cap T)$	
			with hypotheses

	SIMP_CARD_SETMINUS	$\mathrm{card}(S \setminus T) \;\hat{=}\; \mathrm{card}(S) - \mathrm{card}(T)$	$T \subseteq S$ and eit finite(S) or finite(T)
	SIMP_CARD_SETMINUS_SETENUM	$\mathrm{card}(S \setminus \{E_1, \ldots, E_n\}) \;\hat{=}\; \mathrm{card}(S) - \mathrm{card}(\{E_1, \ldots, E_n\})$	with hypotheses $E_i \in S$ for all $i \in 1 \,..\, n$
*	SIMP_CARD_CONVERSE	$\mathrm{card}(r^{-1}) \;\hat{=}\; \mathrm{card}(r)$	
*	SIMP_CARD_ID	$\mathrm{card}(\mathrm{id}) \;\hat{=}\; \mathrm{card}(S)$	where id has ty $\mathbb{P}(S \times S)$
*	SIMP_CARD_ID_DOMRES	$\mathrm{card}(S \lhd \mathrm{id}) \;\hat{=}\; \mathrm{card}(S)$	
*	SIMP_CARD_PRJ1	$\mathrm{card}(\mathrm{prj}_1) \;\hat{=}\; \mathrm{card}(S \times T)$	where prj_1 has type $\mathbb{P}(S \times T \times$
*	SIMP_CARD_PRJ2	$\mathrm{card}(\mathrm{prj}_2) \;\hat{=}\; \mathrm{card}(S \times T)$	where prj_2 has type $\mathbb{P}(S \times T \times$
*	SIMP_CARD_PRJ1_DOMRES	$\mathrm{card}(E \lhd \mathrm{prj}_1) \;\hat{=}\; \mathrm{card}(E)$	
*	SIMP_CARD_PRJ2_DOMRES	$\mathrm{card}(E \lhd \mathrm{prj}_2) \;\hat{=}\; \mathrm{card}(E)$	
*	SIMP_CARD_LAMBDA	$\mathrm{card}(\{x \cdot P \mid E \mapsto F\}) \;\hat{=}\; \mathrm{card}(\{x \cdot P \mid E\})$	where E is a maplet combina of bound identif and expressions that are not bou by the comprehension (i.e., E is syntactically injective) and al identifiers boun the comprehens set that occur in also occur in E
*	SIMP_LIT_CARD_UPTO	$\mathrm{card}(i \,..\, j) \;\hat{=}\; j - i + 1$	where i and j a literals and $i \leq$
	SIMP_TYPE_CARD	$\mathrm{card}(Tenum) \;\hat{=}\; N$	where $Tenum$ carrier set containing N elements
*	SIMP_LIT_GE_CARD_1	$\mathrm{card}(S) \geq 1 \;\hat{=}\; \neg\, S = \varnothing$	
*	SIMP_LIT_LE_CARD_1	$1 \leq \mathrm{card}(S) \;\hat{=}\; \neg\, S = \varnothing$	
*	SIMP_LIT_LE_CARD_0	$0 \leq \mathrm{card}(S) \;\hat{=}\; \top$	
*	SIMP_LIT_GE_CARD_0	$\mathrm{card}(S) \geq 0 \;\hat{=}\; \top$	
*	SIMP_LIT_GT_CARD_0	$\mathrm{card}(S) > 0 \;\hat{=}\; \neg\, S = \varnothing$	
*	SIMP_LIT_LT_CARD_0	$0 < \mathrm{card}(S) \;\hat{=}\; \neg\, S = \varnothing$	
*	SIMP_LIT_EQUAL_CARD_1	$\mathrm{card}(S) = 1 \;\hat{=}\; \exists x \cdot S = \{x\}$	
*	SIMP_CARD_NATURAL	$\mathrm{card}(S) \in \mathbb{N} \;\hat{=}\; \top$	
*	SIMP_CARD_NATURAL1	$\mathrm{card}(S) \in \mathbb{N}_1 \;\hat{=}\; \neg\, S = \varnothing$	
*	SIMP_LIT_IN_NATURAL	$i \in \mathbb{N} \;\hat{=}\; \top$	where i is a non negative literal
*	SIMP_SPECIAL_IN_NATURAL1	$0 \in \mathbb{N}_1 \;\hat{=}\; \bot$	
*	SIMP_LIT_IN_NATURAL1	$i \in \mathbb{N}_1 \;\hat{=}\; \top$	where i is a positive literal
*	SIMP_LIT_UPTO	$i \,..\, j \;\hat{=}\; \varnothing$	where i and j a literals and $j <$
*	SIMP_LIT_IN_MINUS_NATURAL	$-i \in \mathbb{N} \;\hat{=}\; \bot$	where i is a positive literal
*	SIMP_LIT_IN_MINUS_NATURAL1	$-i \in \mathbb{N}_1 \;\hat{=}\; \bot$	where i is a non negative literal
*	DEF_IN_NATURAL	$x \in \mathbb{N} \;\hat{=}\; 0 \leq x$	

*	DEF_IN_NATURAL1	$x \in \mathbb{N}_1 \quad \widehat{=} \quad 1 \leq x$	
*	SIMP_LIT_EQUAL_KBOOL_TRUE	$\mathrm{bool}(P) = \mathrm{TRUE} \quad \widehat{=} \quad P$	
*	SIMP_LIT_EQUAL_KBOOL_FALSE	$\mathrm{bool}(P) = \mathrm{FALSE} \quad \widehat{=} \quad \neg P$	
	DEF_EQUAL_MIN	$E = \min(S) \quad \widehat{=} \quad E \in S \land (\forall x \cdot x \in S \Rightarrow E \leq x)$	where x non fre S, E
	DEF_EQUAL_MAX	$E = \max(S) \quad \widehat{=} \quad E \in S \land (\forall x \cdot x \in S \Rightarrow E \geq x)$	where x non fre S, E
*	SIMP_SPECIAL_PLUS	$E + \ldots + 0 + \ldots + F \quad \widehat{=} \quad E + \ldots + F$	
*	SIMP_SPECIAL_MINUS_R	$E - 0 \quad \widehat{=} \quad E$	
*	SIMP_SPECIAL_MINUS_L	$0 - E \quad \widehat{=} \quad -E$	
*	SIMP_MINUS_MINUS	$-(-E) \quad \widehat{=} \quad E$	
*	SIMP_MINUS_UNMINUS	$E - (-F) \quad \widehat{=} \quad E + F$	where $(-F)$ is unary minus expression or a negative literal
*	SIMP_MULTI_MINUS	$E - E \quad \widehat{=} \quad 0$	
*	SIMP_MULTI_MINUS_PLUS_L	$(A + \ldots + C + \ldots + B) - C \quad \widehat{=} \quad A + \ldots + B$	
*	SIMP_MULTI_MINUS_PLUS_R	$C - (A + \ldots + C + \ldots + B) \quad \widehat{=} \quad -(A + \ldots + B)$	
*	SIMP_MULTI_MINUS_PLUS_PLUS	$(A+\ldots+E+\ldots+B)-(C+\ldots+E+\ldots+D) \quad \widehat{=} \quad (A+\ldots+B)-(C+\ldots+D)$	
*	SIMP_MULTI_PLUS_MINUS	$(A + \ldots + D + \ldots + (C - D) + \ldots + B) \quad \widehat{=} \quad A + \ldots + C + \ldots + B$	
*	SIMP_MULTI_ARITHREL_PLUS_PLUS	$A+\ldots+E+\ldots+B < C+\ldots+E+\ldots+D \quad \widehat{=} \quad A+\ldots+B < C+\ldots+D$	where the root relation ($<$ here one of $\{=, <, \leq, >,$
*	SIMP_MULTI_ARITHREL_PLUS_R	$C < A + \ldots + C \ldots + B \quad \widehat{=} \quad 0 < A + \ldots + B$	where the root relation ($<$ here one of $\{=, <, \leq, >,$
*	SIMP_MULTI_ARITHREL_PLUS_L	$A + \ldots + C \ldots + B < C \quad \widehat{=} \quad A + \ldots + B < 0$	where the root relation ($<$ here one of $\{=, <, \leq, >,$
*	SIMP_MULTI_ARITHREL_MINUS_MINUS_R	$A - C < B - C \quad \widehat{=} \quad A < B$	where the root relation ($<$ here one of $\{=, <, \leq, >,$
*	SIMP_MULTI_ARITHREL_MINUS_MINUS_L	$C - A < C - B \quad \widehat{=} \quad B < A$	where the root relation ($<$ here one of $\{=, <, \leq, >,$
*	SIMP_SPECIAL_PROD_0	$E * \ldots * 0 * \ldots * F \quad \widehat{=} \quad 0$	
*	SIMP_SPECIAL_PROD_1	$E * \ldots * 1 * \ldots * F \quad \widehat{=} \quad E * \ldots * F$	
*	SIMP_SPECIAL_PROD_MINUS_EVEN	$(-E) * \ldots * (-F) \quad \widehat{=} \quad E * \ldots * F$	if an even numb of $-$
*	SIMP_SPECIAL_PROD_MINUS_ODD	$(-E) * \ldots * (-F) \quad \widehat{=} \quad -(E * \ldots * F)$	if an odd numbe $-$
*	SIMP_LIT_MINUS	$-(i) \quad \widehat{=} \quad (-i)$	where i is a liter
*	SIMP_LIT_EQUAL	$i = j \quad \widehat{=} \quad \top \; or \; \bot \; (computation)$	where i and j a literals
*	SIMP_LIT_LE	$i \leq j \quad \widehat{=} \quad \top \; or \; \bot \; (computation)$	where i and j a literals
*	SIMP_LIT_LT	$i < j \quad \widehat{=} \quad \top \; or \; \bot \; (computation)$	where i and j a literals
*	SIMP_LIT_GE	$i \geq j \quad \widehat{=} \quad \top \; or \; \bot \; (computation)$	where i and j a literals
*	SIMP_LIT_GT	$i > j \quad \widehat{=} \quad \top \; or \; \bot \; (computation)$	where i and j a literals

*	SIMP_DIV_MINUS	$(-E) \div (-F) \;\; \widehat{=} \;\; E \div F$	
*	SIMP_SPECIAL_DIV_1	$E \div 1 \;\; \widehat{=} \;\; E$	
*	SIMP_SPECIAL_DIV_0	$0 \div E \;\; \widehat{=} \;\; 0$	
*	SIMP_SPECIAL_EXPN_1_R	$E^1 \;\; \widehat{=} \;\; E$	
*	SIMP_SPECIAL_EXPN_1_L	$1^E \;\; \widehat{=} \;\; 1$	
*	SIMP_SPECIAL_EXPN_0	$E^0 \;\; \widehat{=} \;\; 1$	
*	SIMP_MULTI_LE	$E \leq E \;\; \widehat{=} \;\; \top$	
*	SIMP_MULTI_LT	$E < E \;\; \widehat{=} \;\; \bot$	
*	SIMP_MULTI_GE	$E \geq E \;\; \widehat{=} \;\; \top$	
*	SIMP_MULTI_GT	$E > E \;\; \widehat{=} \;\; \bot$	
*	SIMP_MULTI_DIV	$E \div E \;\; \widehat{=} \;\; 1$	
*	SIMP_MULTI_DIV_PROD	$(X * \ldots * E * \ldots * Y) \div E \;\; \widehat{=} \;\; X * \ldots * Y$	
*	SIMP_MULTI_MOD	$E \bmod E \;\; \widehat{=} \;\; 0$	
	DISTRI_PROD_PLUS	$a * (b + c) \;\; \widehat{=} \;\; (a * b) + (a * c)$	
	DISTRI_PROD_MINUS	$a * (b - c) \;\; \widehat{=} \;\; (a * b) - (a * c)$	
	DERIV_NOT_EQUAL	$\neg E = F \;\; \widehat{=} \;\; E < F \lor E > F$	E and F must of Integer type

Extension Proof Rules

Rules that are marked with a * in the first column are implemented in the latest version of Rodin. Rules without a * are planned to be implemented in future versions. Other conventions used in these tables are described in The_Proving_Perspective_(Rodin_User_Manual)#Rewrite_Rules.

Rewrite Rules

	Name	Rule	Side Condition	A/M
*	SIMP_SPECIAL_COND_BTRUE	$\mathrm{COND}(\top, E_1, E_2) \;\; \widehat{=} \;\; E_1$		A
*	SIMP_SPECIAL_COND_BFALSE	$\mathrm{COND}(\bot, E_1, E_2) \;\; \widehat{=} \;\; E_2$		A
*	SIMP_MULTI_COND	$\mathrm{COND}(C, E, E) \;\; \widehat{=} \;\; E$		A

Inference Rules

	Name	Rule	Side Condition
*	DATATYPE_DISTINCT_CASE	$\dfrac{\mathbf{H}, x = c_1(p_{11}, \ldots, p_{1k}) \;\vdash\; \mathbf{G} \qquad \ldots \qquad \mathbf{H}, x = c_n(p_{n1}, \ldots, p_{nl}) \;\vdash\; \mathbf{G}}{\mathbf{H} \;\vdash\; \mathbf{G}}$	where x has a datatype DT as typ and appears free in $\mathbf{G}$, DT has constructors $c_1, \ldots, c_n$, parameters p_{ij} are introduced as fresh identifiers
*	DATATYPE_INDUCTION	$\dfrac{\mathbf{H}, x = c_1(p_1, \ldots, p_k), \mathbf{P}(p_{I_1}), \ldots, \mathbf{P}(p_{I_l}) \;\vdash\; \mathbf{P}(x) \qquad \ldots}{\mathbf{H} \;\vdash\; \mathbf{P}(x)}$	where x has inductive datatype DT as type and appears free in $\mathbf{P}$ $\{p_{I_1}, \ldots, p_{I_l}\} \subseteq \{p_1, \ldots, p_k$ are the inductive parameters (if any); an antecedent is created for every constructor c_i of DT; all parameters are introduced as fresh identifiers; examples here

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